Super Mental Addition 2D

Tower Totals

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The teacher puts 4 rectangular towers of 4 numbers like the ones shown below on the desk. Each of the number towers has a back side that also contains 4 numbers.

<u> </u>	set of 4	Fronts	
9	3	7	7
8	6	3	5
5	6	4	9
4	9	7	2

9

8

5

4

8

3

6

6

9

6

7

3

4

7

3

7

5

9

2

3



Has this set of 4 Backs

A student is asked to place the 4 number towers on the desk next to each other The 4 towers can be placed in any order and turned over so that any of the sides are facing up or down. The 4 towers form 4 rows of numbers. You can instantly state the sum of the 4 numbers on the front AND ON THE BACK

Note: To check the sum of the numbers on the back you will need to flip over the towers as one group. be sure to hold the towers together and flip them **right to left as one group.** Do not just turn each tower over in place. It is easier to flip over 2 or 3 towers then all 4

The technique used to find the sum of the 4 numbers on the front and back works for 1, 2, 3 or all 4 towers but it is more impressive the more towers that are used.

TOTAL = 2

Patterns for the 4 Towers



Note: Two larger patterns are provided on pages 6, 7 and 8

Preparation:

Cut out each tower. Fold the front and back together so that the numbers show on the front and back and then tape the towers at the seam.

Procedure

Put the 4 towers on a desk. Turn one or 2 towers over so the student can see that there are also 4 numbers on the other side. Show the student that you can put the 4 towers next to each other to form 4 numbers in 4 rows. After you have demonstrated this, hand the towers to the student. and ask them to place all or some of the 4 towers in any order they like. After you verify they have placed the towers as requested have the student write down the numbers on a sheet of paper and then add up the four 4 digit numbers. If you want them to use a calculator that would be fine. As soon as they start to add the numbers write the sum of the front faces on a small piece of paper and label that number the total of front numbers. Than write down the sum of the back faces on the piece of paper and label that number the total of back numbers. Fold the paper and put it on the desk. When the student has found the sum of the numbers on the front check to be sure they are correct. Take the 4 towers as one group and flip them over right to left. Ask them to find the sum of those four 4 digit numbers. When they are done check to be sure they are correct. Tell the student you have written the sum of the numbers on the front and the sum of the numbers on the back as soon as they started to add the numbers. Emphasize that you did not see the numbers on the back before you wrote done your answers. Have them open the paper on the desk and verify that you were able to find that correct sums faster then they did and without seeing the back side of the towers.

You can allow the student to choose as many of the towers as they want to use. The technique to find the total works for 1, 2, 3 or all 4 towers bu it is more impressive the more towers you use.

NOTE: Some students will see some patterns right away. Others may be able to state some facts about the sums after seeing a few repetitions of the trick. Most magicians do not repeat tricks because they do not want the audience to get any idea how a trick works. You, on the other hand, are a teacher. You want students to observe an effect several times and begin to make conjectures. If a student can figure out how an effect works and tell you that is great. If they can write out the procedure for finding the sum that is even better. If they can explain how the numbers were chosen that would be the even better. If they could then create a new set of numbers that also work that would be a crowning achievement. The only way these things can happen is by having them see the effect several times and record their observations, They also need to look at the numbers on the towers and note any observations. I do not do tricks to amaze or entertain. I do them as a starting point to motivate students to observe, record and organize data. I hope to lead them part way to the solution by asking leading questions that help them build on their initial conclusions.

Almost every student can make a few initial observations and many will find the way the sum is found. If any student wants to dig deeper I am happy to give them a printout of the numbers and let them make their one towers and keep working to see if they can explain how the numbers were chosen and maybe even make a new set with different numbers. Only a very few will go that far but for any that do they will remember that moment they found the key idea for the rest of their lives. Everybody will get something from the activity and some students may get a great deal more.

Find the total of the numbers on the FRONT FACES of 4 towers.

The Second ROW of the front is used to find the sum of the numbers on the FRONT FACES.

- A. Say the number 2
- B. Read the next 3 numbers on the **Second Row** from **LEFT TO RiGHT**.
- C. The last digit of the sum will be 2 less than the number on the RIGHT END of the row.



Find the total of the numbers on the BACK FACES of 4 towers.

The Last Row of the front is used to find the sum of the numbers on the BACK FACES.

These are the number on the front faces. To verify the sum of the numbers on the back flip the towers over as a group form right to left



- A. Say the number 2.
- B. Read the next 3 numbers from the Last ROW from RIGHT TO LEFT.
- C. The last digit of the sum will be 2 less than the number on the LEFT END of the row.
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Find the total of the numbers on the FRONT FACES of 3 prisms.

The Second ROW of the front is used to find the sum of the numbers on the FRONT FACES.

- A. Say the number 2
- B. Read the next 2 numbers on the **Second Row** from **LEFT TO RiGHT**.
- C. The last digit of the sum will be 2 less than the number on the RIGHT END of the row.



Find the total of the numbers on the BACK FACES of 3 towers.

The Last Row of the front is used to find the sum of the numbers on the BACK FACES.

These are the number on the front faces to verify the sum of the numbers on the back flip the towers over as a group form right to left



TOTAL on back will be = 2 4 2 7

- A. Say the number 2.
- B. Read the next 2 numbers from the Last ROW from RIGHT TO LEFT.
- C. The last digit of the sum will be 2 less than the number on the LEFT END of the row.

Set A

Student Size Patterns for 4 Towers with 4 rows Carry and Difference are 2, Front Second Row, Back Last Row

These towers are based on the second row predicting the sum of the front numbers and the last row predicting the sum of the back numbers. It has the number 2 as the number carried over and the difference between the digit on the tower and the ones digit of the sum is 2.

			Se	et A			
Tower	A 1	Tower	A2	Tower	A 3	Tower	A 4
9	6	3	7	7	9	7	5
8	4	6	9	3	7	5	2
5	4	6	5	4	6	9	8
4	8	9	6	7	3	2	5

Student Size Patterns for the 4 Towers with 4 rows Carry and Difference are 2, Front Second Row, Back Last Row

These towers are based on the second row predicting the sum of the front numbers and the last row predicting the sum of the back numbers. It has the number 2 as the number carried over and the difference between the digit on the tower and the ones digit of the sum is 2.



Tower	A3	Tower	A4
7	9	7	5
3	7	5	2
4	6	9	8
7	3	2	5

How to create the numbers for the A1 towers Set A

Towe front -	r A1 - back	Towe front	er A2 <u>– back</u>	 Tower front -	A3 - back	Towe front -	er A4 - back
9	6	3	7	7	9	7	5
8	4	6	9	3	7	5	2
5	4	6	5	4	6	9	8
4	8	9	6	7	3	2	5

Of all the mathematical effects I have worked with to date, this "trick" is the most unusual. **There is no trick.** You just decide on the parameters of the rules and then put the numbers on the faces to meet those requirements. You really just "make the trick work"

- Step 1. Pick how many numbers you want in each tower. I picked 4
- Step 2. Pick a row for the number that revels the sum on the front. I picked the second row.
- Step 3. Pick a row for the number that revels the sum on the back. I picked the bottom row.
- Step 4. The number in the row you selected in step 2 will be larger than the ones digit of the sum of the column. Pick how much larger it will be. I picked 2. The number is row 2 will be 2 more than the ones digit for the column total. The totals of each tower must be from 20 to 27. If you had picked 1 the totals of each tower would need to be from 10 to 18

The next several pages describe this process for the set of towers shown below. I suggest you print a blank set of towers and work along with the example. It is the best way to understand the process.



Create a tower with a total of 26 on the front side and a total of 22 on the back side.

Create the front side that has a total of 26.

Step 1. We picked this side to total 26. We decided the number in the second row would be 2 more than the ones digit of the column total. The number in the second row must be 2 more than 6 It must be 8.

8	

Step 2. We picked the back of this tower total 22. The number in the last row on the front must be 2 more than the ones digit of the total on back. The number in last row must be 2 more than 2 It must be 4

8	
4	

Create the side that has a total of 22.

Step 1. We picked this side to total 2. We decided the number in the second row would be 2 more than the ones digit of the column total. The number in the second row must be 2 more than 2 It must be 4.



Step 2. We picked the back of this tower total 26. The number in the last row on the front must be 2 more than the ones digit of the total on back. The number in last row must be 2 more than 6 It must be 8

4	
8	

Step 3. We picked this side to total 26. Row 2 and row 4 total 12. The the remaining rows 1 and row 3 must total 14. Any 2 numbers that total 14 can go in row 1 and row 3. row 1 chose 9 and 5



Step 3. We picked this side to total 22. Row 2 and row 4 total 12. The the remaining rows 1 and row 3 must total 10. Any 2 numbers that total 10 can go in row 1 and row 3. row I chose 6 and 4

6	
4	
4	
8	

Create a tower with a total of 24 on the front side and a total of 27 on the back side.

Create the front side that has a total of 24.

Step 1. We picked this side to total 24. We decided the number in the second row would be 2 more than the ones digit of the column total. The number in the second row must be 2 more than 4 It must be 6.

6	

Step 2. We picked the back of this tower total 27. The number in the last row on the front must be 2 more than the ones digit of the total on back. The number in last row must be 2 more than 7 It must be 9

6	
9	

Create the side that has a total of 22.

Step 1. We picked this side to total 2. We decided the number in the second row would be 2 more than the ones digit of the column total. The number in the second Row must be 2 more than 2 It must be 4.



Step 2. We picked the back of this tower total 26. The number in the last row on the front must be 2 more than the ones digit of the total on back. The number in last row must be 2 more than 6 It must be 8

4	
8	

Step 3. We picked this side to total 24. Row 2 and row 4 total 15. The the remaining rows 1 and row 3 must total 9. Any 2 numbers that total 9 can go in row 1 and row 3. row 1 chose 3 and 6



Step 3. We picked this side to total 22. Row 2 and row 4 total 12. The the remaining rows 1 and row 3 must total 10. Any 2 numbers that total 10 can go in row 1 and row 3. I chose 6 and 4

6	
4	
4	
8	

Create a tower with a total of 21 on the front side and a total of 25 on the back side.

Create the front side that has a total of 21.

Step 1. We picked this side to total 21. We decided the number in the second row would be 2 more than the ones digit of the column total. The number in the second Row must be 2 more than 1 It must be 3.

3	

Step 2. We picked the back of this tower total 25. The number in the last row on the front must be 2 more than the ones digit of the total on back. The number in last row must be 2 more than 5 It must be 7

3	
7	

Create the side that has a total of 25.

Step 1. We picked this side to total 25. We decided the number in the second row would be 2 more than the ones digit of the column total. The number in the second row must be 2 more than 5 It must be 7.



Step 2. We picked the back of this tower total 21. The number in the last row on the front must be 2 more than the ones digit of the total on back. The number in last row must be 2 more than 1 It must be 3

7	
3	

Step 3. We picked this side to total 21. Row 2 and row 4 total 10. The the remaining rows 1 and row 3 must total 11. Any 2 numbers that total 11 can go in row 1 and row 3. I chose 7 and 4



Step 3. We picked this side to total 25. Row 2 and row 4 total 10. The the remaining rows 1 and row 3 must total 15. Any 2 numbers that total 15 can go in row 1 and row 3. I chose 9 and 6

9	
7	
6	
3	

Create a tower with a total of 23 on the front side and a total of 20 on the back side.

Create the front side that has a total of 21.

Step 1. We picked this side to total 23. We decided the number in the second row would be 2 more than the ones digit of the column total. The number in the second Row must be 2 more than 3 It must be 5.

5	

Step 2. We picked the back of this tower total 20. The number in the last row on the front must be 2 more than the ones digit of the total on back. The number in last row must be 2 more than 0 It must be 2

5	
2	

Create the side that has a total of 20.

Step 1. We picked this side to total 20. We decided the number in the second row would be 2 more than the ones digit of the column total. The number in the second row must be 2 more than 0 It must be 2.



Step 2. We picked the back of this tower total 23. The number in the last row on the front must be 2 more than the ones digit of the total on back. The number in last row must be 2 more than 3 It must be 5

2	
5]

Step 3. We picked this side to total 23. Row 2 and row 4 total 7. The the remaining rows 1 and row 3 must total 16. Any 2 numbers that total 16 can go in row 1 and row 3. I chose 7 and 9.



Step 3. We picked this side to total 20. Row 2 and row 4 total 7. The the remaining rows 1 and row 3 must total 13. Any 2 numbers that total 13 can go in row 1 and row 3. I chose 5 and 8

5	
2	
8	
5	

Student Size Patterns for 4 Towers with 4 rows Carry and Difference 1, Front Second Row, Back Last Row

These towers are based on the second row predicting the sum of the front numbers and the last row predicting the sum of the back numbers. It has the number 1 as the number carried over and the difference between the digit on the tower and the ones digit of the sum is 1.



NOTE: I have provided 4 different sets of basic towers. Each of the sets use a different number and a different rule. If you use more than one set in the classroom be sure to print the different sets on **different colors of paper.** This will stop the students (or you) from mixing up the different sets of towers and creating a disaster. You could also leave the title on the top of the tower when you cut it out. That is what I do.

You may decide to leave the bottom totals on a set of towers after you have done the trick and are working to get the students to see how the effect works. Seeing the separate totals may help them see how the carrying works.

Student Size Patterns for 4 Towers with 5 rows Carry and Difference 1, Front Second Row, Back Last Row

These towers are based on the second row predicting the sum of the front numbers and the last row predicting the sum of the back numbers. It has the number 1 as the number carried over and the difference between the digit on the tower and the ones digit of the sum is 1.



NOTE: I have provided 4 different sets of basic towers. Each of the sets use a different number and a different rule. If you use more than one set in the classroom be sure to print the different sets on **different colors of paper.** This will stop the students (or you) from mixing up the different sets of towers and creating a disaster. You could also leave the title on the top of the tower when you cut it out. That is what I do.

You may decide to leave the bottom totals on a set of towers after you have done the trick and are working to get the students to see how the effect works. Seeing the separate totals may help them see how the carrying works.

Student Size Patterns for 4 Towers with 5 rows Carry and Difference 2, Front Second Row, Back Last Row

These towers are based on the second row predicting the sum of the front numbers and the last row predicting the sum of the back numbers. It has the number 2 as the number carried over and the difference between the digit on the tower and the ones digit of the sum is 2.



NOTE: I have provided 4 different sets of basic towers. Each of the sets use a different number and a different rule. If you use more than one set in the classroom be sure to print the different sets on **different colors of paper.** This will stop the students (or you) from mixing up the different sets of towers and creating a disaster. You could also leave the title on the top of the tower when you cut it out. That is what I do.

You may decide to leave the bottom totals on a set of towers after you have done the trick and are working to get the students to see how the effect works. Seeing the separate totals may help them see how the carrying works.



A set of towers using a slightly different rule

Set E

The total of the numbers on the front of the towers is found using the sums of the first two rows.

Total of the numbers on the front



The total of the numbers on the back of the towers is found using the sums of the last two rows.

Total of the numbers on the back



