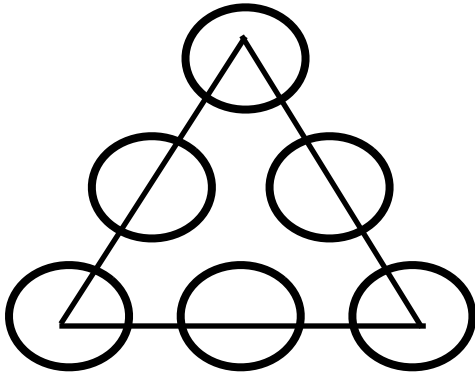
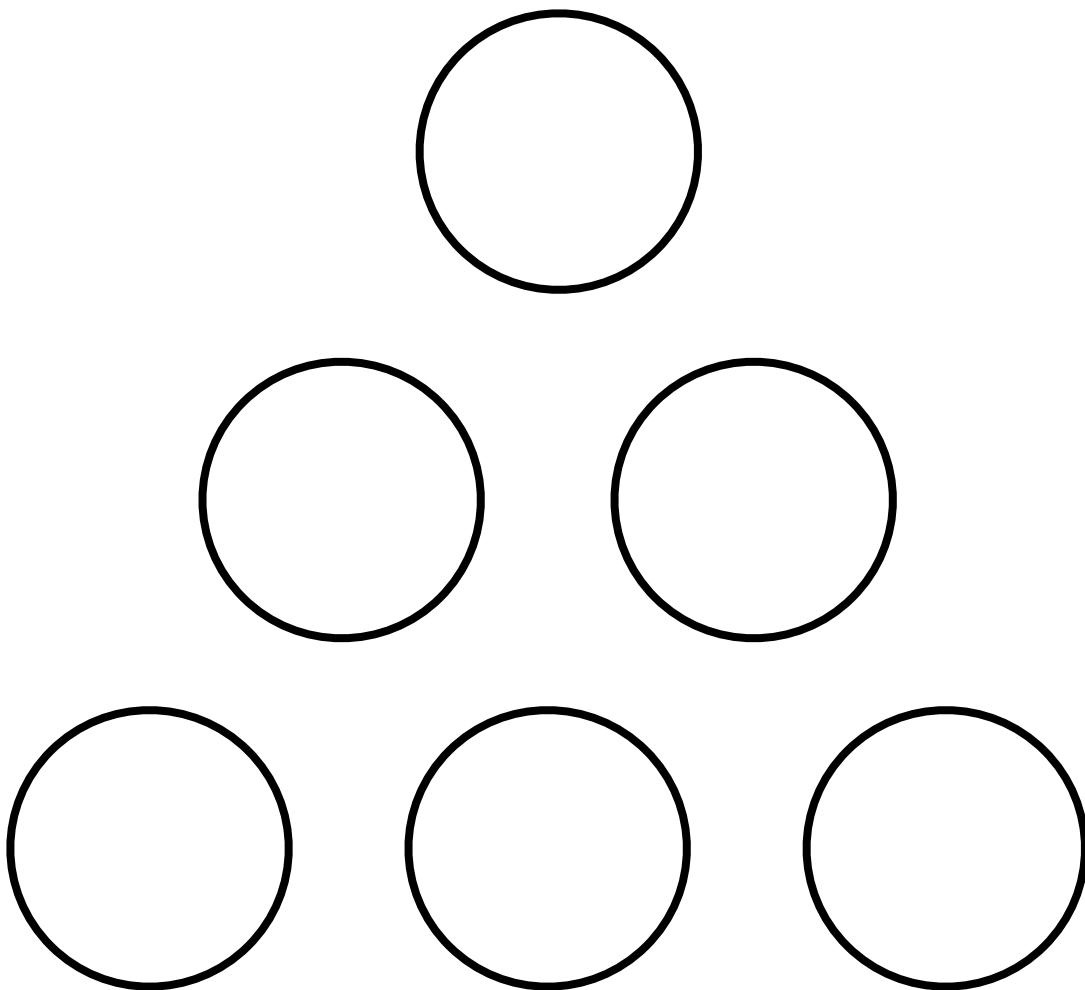


Perimeter Magic Triangles Sum to 9

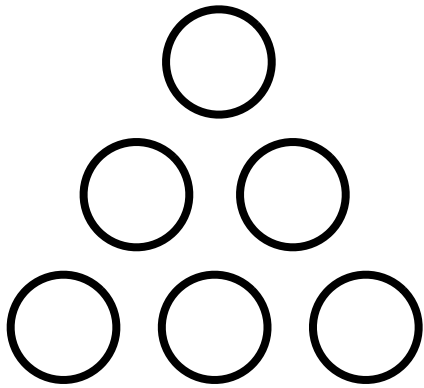


Arrange each of the numbers from 1 to 6
in the triangles
so that the numbers
in each row of 3 triangles
have a **Sum of 9.**

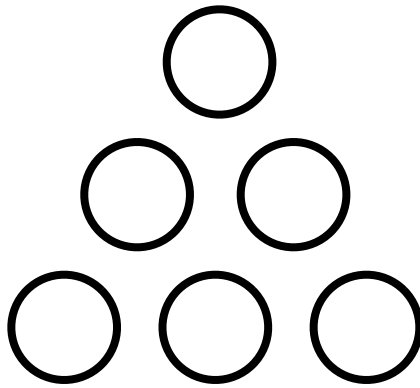


Perimeter Magic Triangles Sum to 9 help page

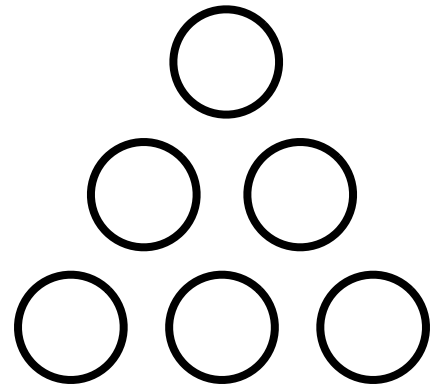
Find 3 **different** numbers from 1 to 6 that have a total of 10 and write them on the bottom row of the first picture. Then use the **remaining 3 numbers** to fill in the last 3 circles. See if the total of each row is 9. If it is not, keep trying different combinations until you find a way to use each of the numbers from 1 to 6 and get all 3 rows to total 9?



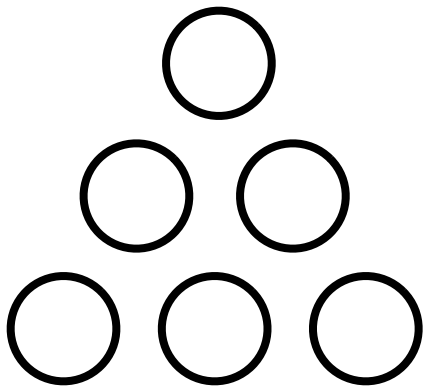
attempt 1



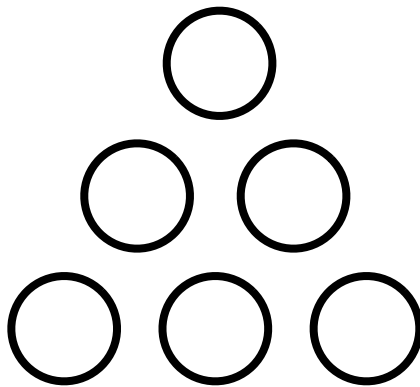
attempt 2



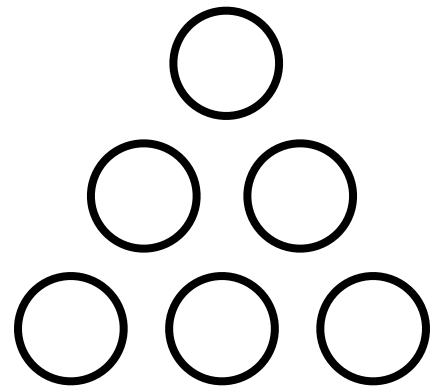
attempt 3



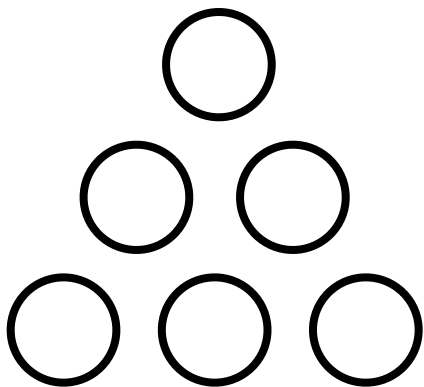
attempt 4



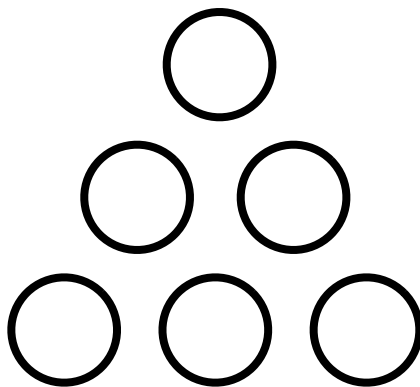
attempt 5



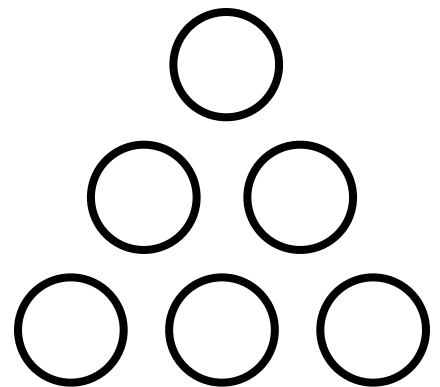
attempt 6



attempt 7



attempt 8



attempt 9

Perimeter Magic Triangle Sum to 9 help page

Find all the different ways that 3 numbers from 1 to 6 can be added to get a **total of 9**. You cannot use the same number twice in any sum. The same 3 numbers can be used in different orders. EXAMPLE: $1 + 2 + 6 = 9$ and $1 + 6 + 2 = 9$ both work.

starts with a 1

$1 + \underline{\quad} + \underline{\quad}$

$1 + \underline{\quad} + \underline{\quad}$

$1 + \underline{\quad} + \underline{\quad}$

$1 + \underline{\quad} + \underline{\quad}$

starts with a 2

$2 + \underline{\quad} + \underline{\quad}$

$2 + \underline{\quad} + \underline{\quad}$

$2 + \underline{\quad} + \underline{\quad}$

$2 + \underline{\quad} + \underline{\quad}$

starts with a 3

$3 + \underline{\quad} + \underline{\quad}$

$3 + \underline{\quad} + \underline{\quad}$

$3 + \underline{\quad} + \underline{\quad}$

$3 + \underline{\quad} + \underline{\quad}$

starts with a 4

$4 + \underline{\quad} + \underline{\quad}$

$4 + \underline{\quad} + \underline{\quad}$

$4 + \underline{\quad} + \underline{\quad}$

$4 + \underline{\quad} + \underline{\quad}$

starts with a 5

$5 + \underline{\quad} + \underline{\quad}$

$5 + \underline{\quad} + \underline{\quad}$

$5 + \underline{\quad} + \underline{\quad}$

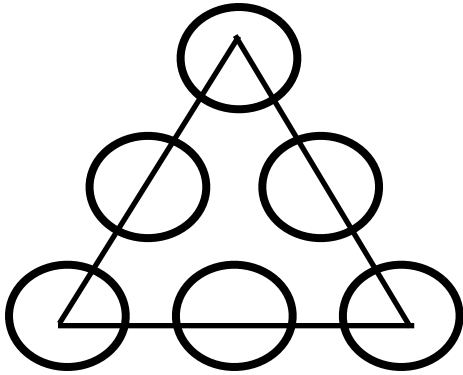
$5 + \underline{\quad} + \underline{\quad}$

starts with a 6

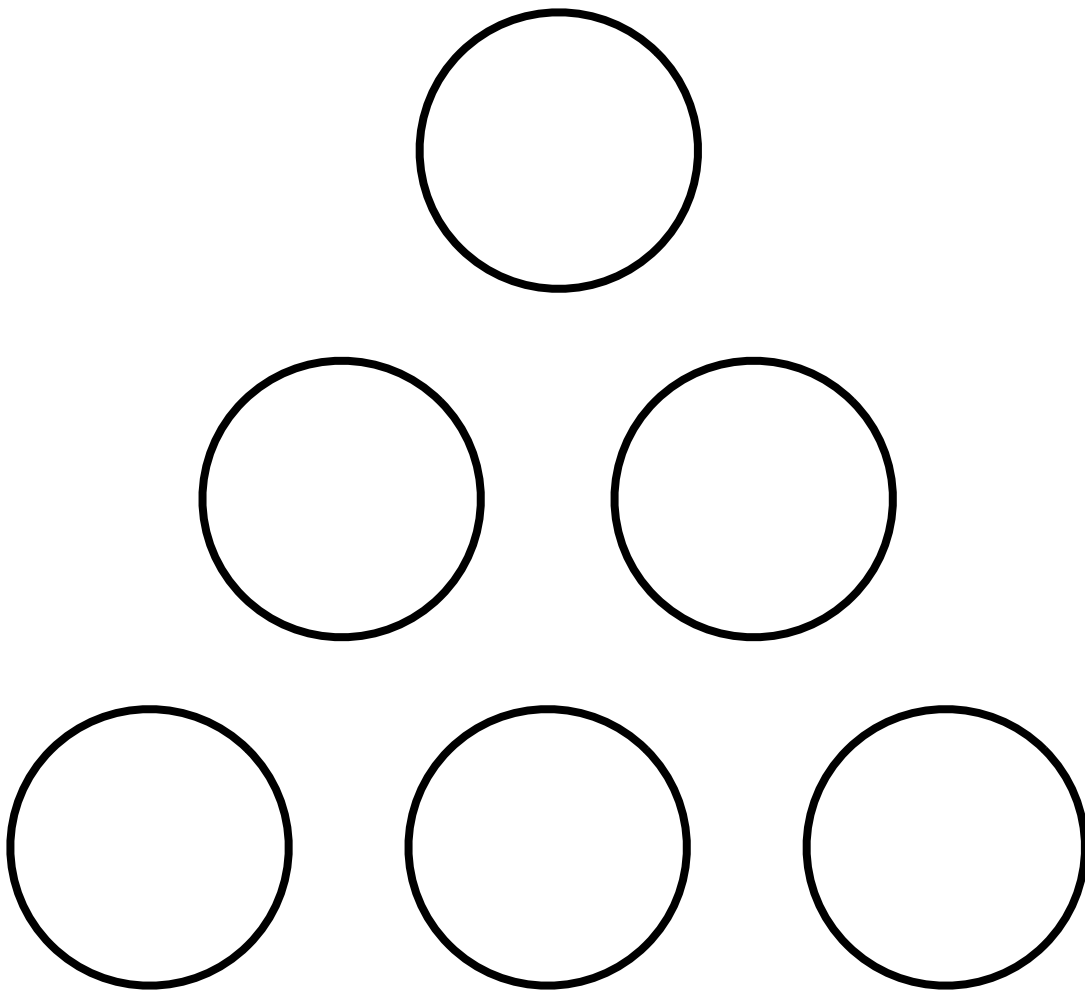
$6 + \underline{\quad} + \underline{\quad}$

$6 + \underline{\quad} + \underline{\quad}$

Perimeter Magic Triangle Sum to 10

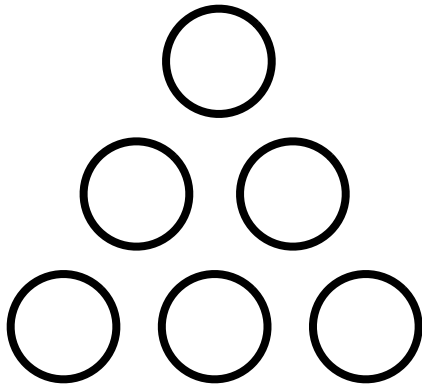


Arrange **each** of the **numbers from 1 to 6**
in the triangles
so that the numbers
in each row of 3 triangles
have a **Sum of 10**.

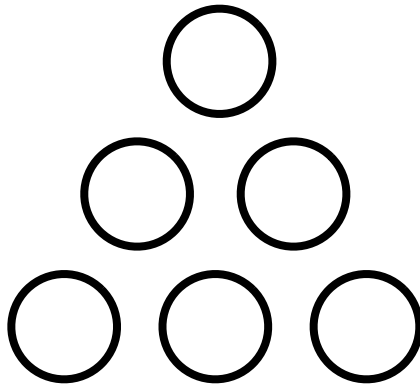


Perimeter Magic Triangle Sum to 10 help page

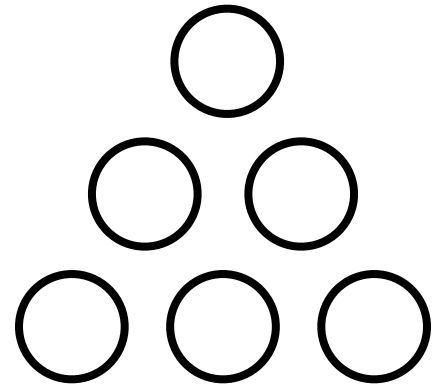
Find 3 **different** numbers from 1 to 6 that have a **total of 10** and write them on the bottom row of the first picture. Then use the **remaining 3 numbers** to fill in the last 3 circles. See if the total of each row is 10. If it is not, keep trying different combinations until you find a way to use each of the numbers from 1 to 6 and get all 3 rows to total 10?



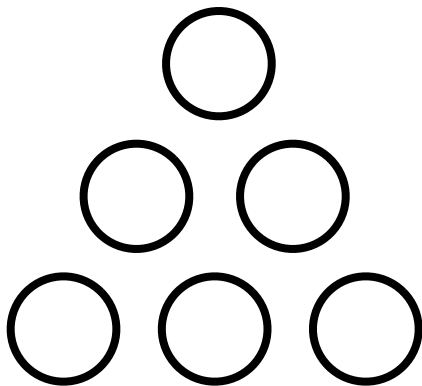
attempt 1



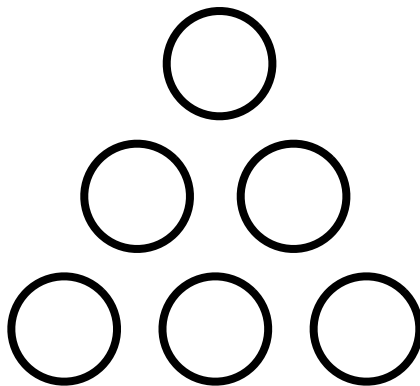
attempt 2



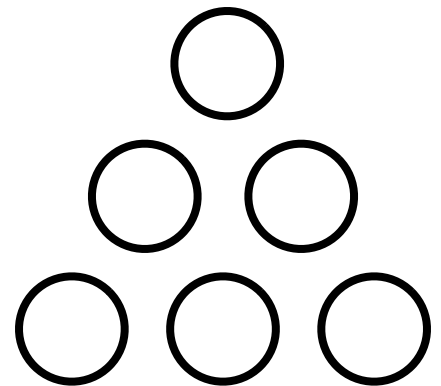
attempt 3



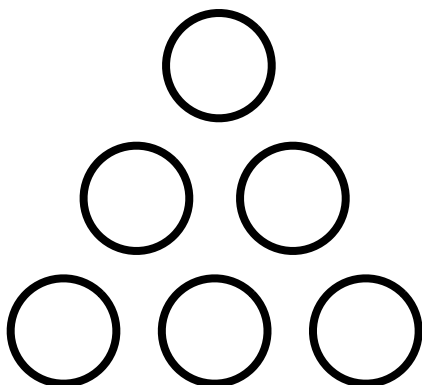
attempt 4



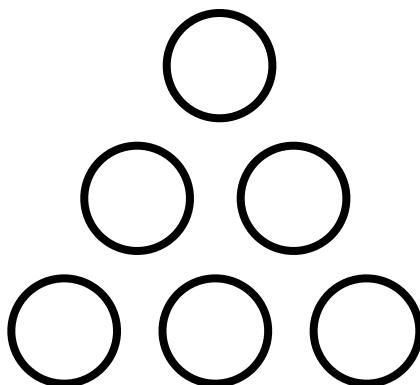
attempt 5



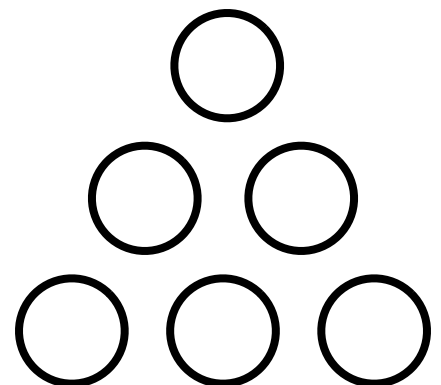
attempt 6



attempt 7



attempt 8



attempt 9

Perimeter Magic Triangle Sum to 10 help page

Find all the different ways that 3 numbers from 1 to 6 can be added to **get a total of 10.**

You cannot use the same number twice in any sum. The same 3 numbers can be used in different orders. EXAMPLE: $1 + 3 + 6 = 10$ and $1 + 6 + 3 = 10$ both work.

starts with a 1

$1 + \underline{\quad} + \underline{\quad}$

$1 + \underline{\quad} + \underline{\quad}$

$1 + \underline{\quad} + \underline{\quad}$

$1 + \underline{\quad} + \underline{\quad}$

starts with a 2

$2 + \underline{\quad} + \underline{\quad}$

$2 + \underline{\quad} + \underline{\quad}$

starts with a 3

$3 + \underline{\quad} + \underline{\quad}$

$3 + \underline{\quad} + \underline{\quad}$

$3 + \underline{\quad} + \underline{\quad}$

$3 + \underline{\quad} + \underline{\quad}$

starts with a 4

$4 + \underline{\quad} + \underline{\quad}$

$4 + \underline{\quad} + \underline{\quad}$

$4 + \underline{\quad} + \underline{\quad}$

$4 + \underline{\quad} + \underline{\quad}$

starts with a 5

$5 + \underline{\quad} + \underline{\quad}$

$5 + \underline{\quad} + \underline{\quad}$

$5 + \underline{\quad} + \underline{\quad}$

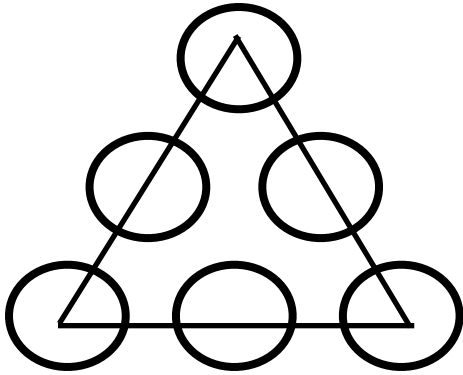
$5 + \underline{\quad} + \underline{\quad}$

starts with a 6

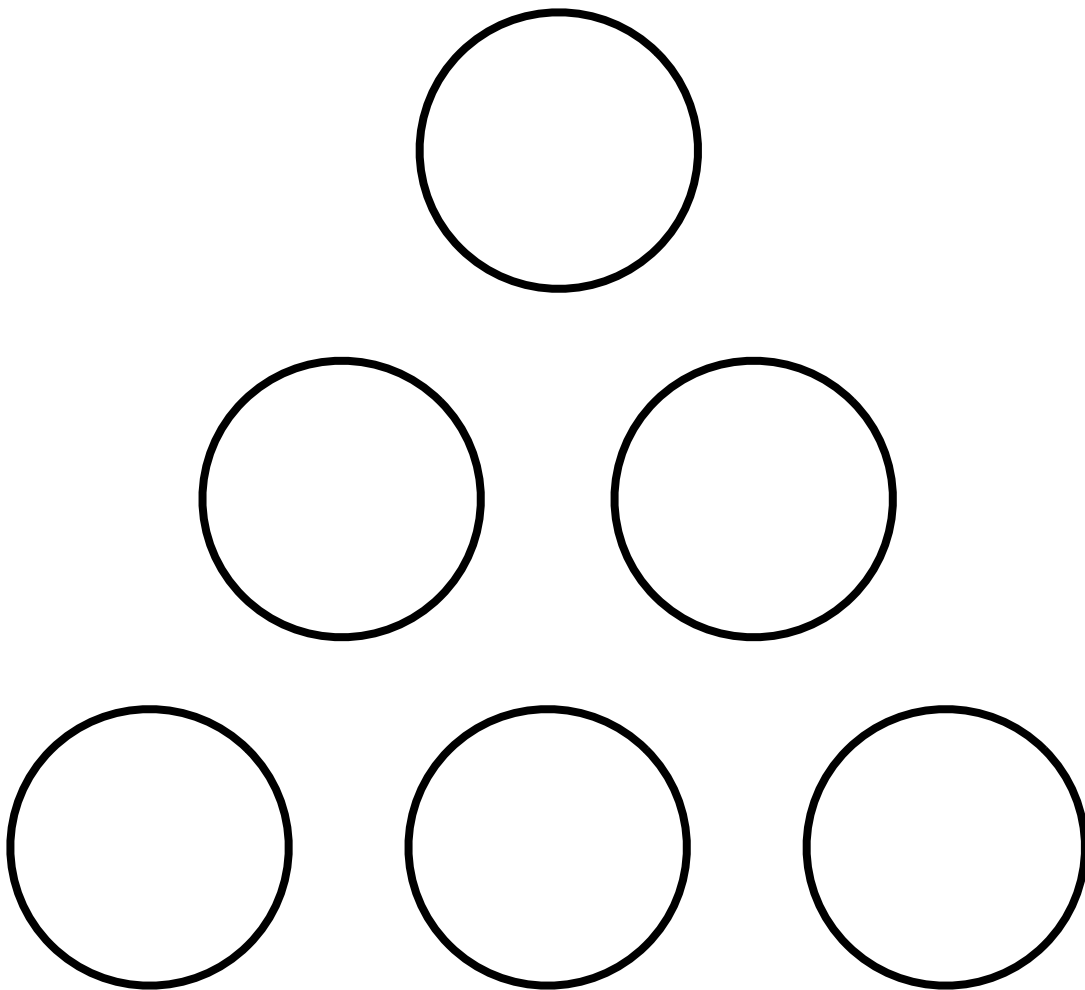
$6 + \underline{\quad} + \underline{\quad}$

$6 + \underline{\quad} + \underline{\quad}$

Perimeter Magic Triangle Sum to 11

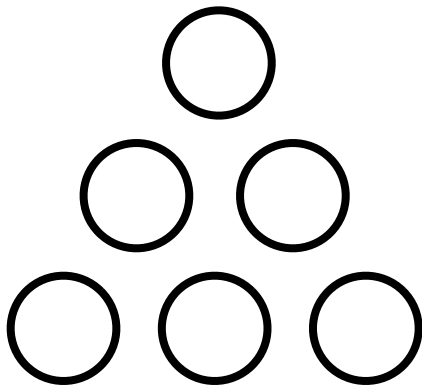


Arrange **each** of the **numbers from 1 to 6**
in the triangles
so that the numbers
in each row of 3 triangles
have a **Sum of 11**.

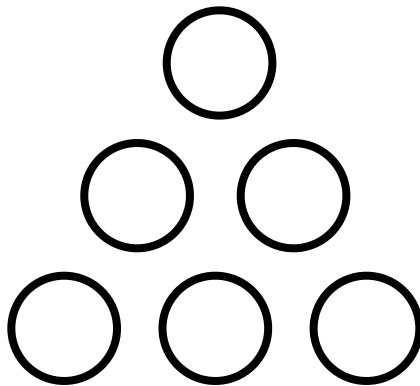


Perimeter Magic Triangle Sum to 11 help page

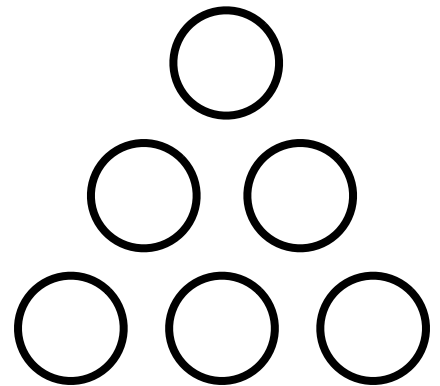
Find 3 **different** numbers from 1 to 6 that have a **total of 11** and write them on the bottom row of the first picture. Then use the **remaining 3 numbers** to fill in the last 3 circles. See if the total of each row is **11**. If it is not, keep trying different combinations until you find a way to use each of the numbers from 1 to 6 and get all 3 rows **to total 11**?



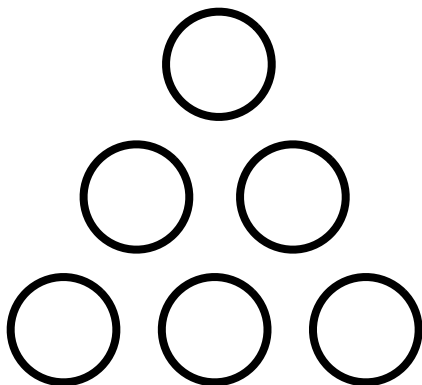
attempt 1



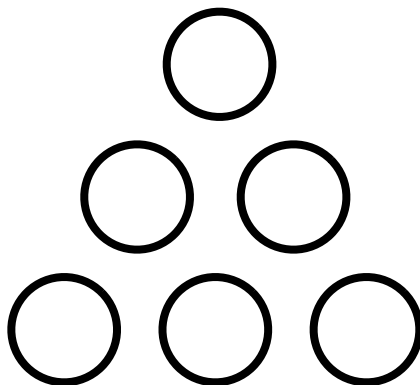
attempt 2



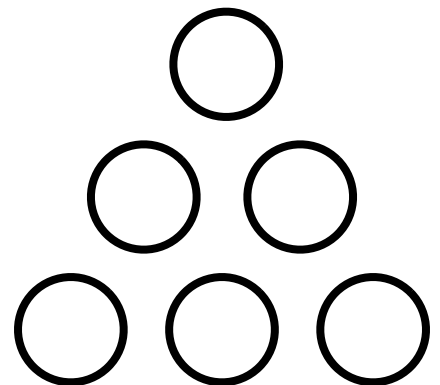
attempt 3



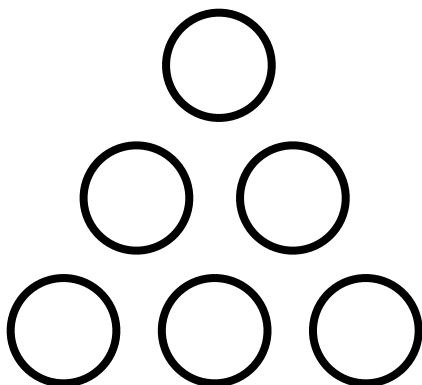
attempt 4



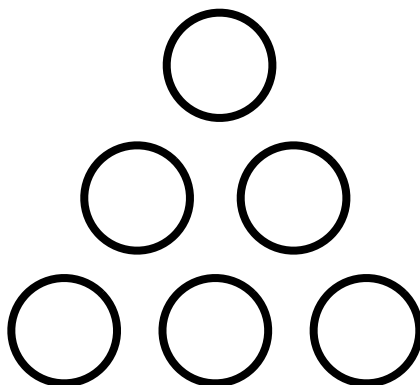
attempt 5



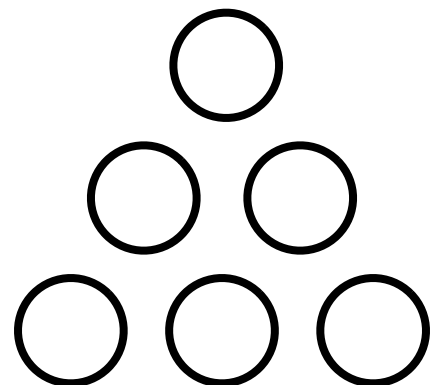
attempt 6



attempt 7



attempt 8



attempt 9

Perimeter Magic Triangle Sum to 11 help page

Find all the different ways that 3 numbers from 1 to 6 can be added to get a **total of 11**.

You cannot use the same number twice in any sum. The same 3 numbers can be used in different orders. EXAMPLE: $1 + 4 + 6 = 11$ and $1 + 6 + 4 = 11$ both work.

starts with a 1

$$1 + \underline{\quad} + \underline{\quad}$$

$$1 + \underline{\quad} + \underline{\quad}$$

starts with a 2

$$2 + \underline{\quad} + \underline{\quad}$$

$$2 + \underline{\quad} + \underline{\quad}$$

$$2 + \underline{\quad} + \underline{\quad}$$

$$2 + \underline{\quad} + \underline{\quad}$$

starts with a 3

$$3 + \underline{\quad} + \underline{\quad}$$

$$3 + \underline{\quad} + \underline{\quad}$$

starts with a 4

$$4 + \underline{\quad} + \underline{\quad}$$

$$4 + \underline{\quad} + \underline{\quad}$$

$$4 + \underline{\quad} + \underline{\quad}$$

$$4 + \underline{\quad} + \underline{\quad}$$

starts with a 5

$$5 + \underline{\quad} + \underline{\quad}$$

$$5 + \underline{\quad} + \underline{\quad}$$

starts with a 6

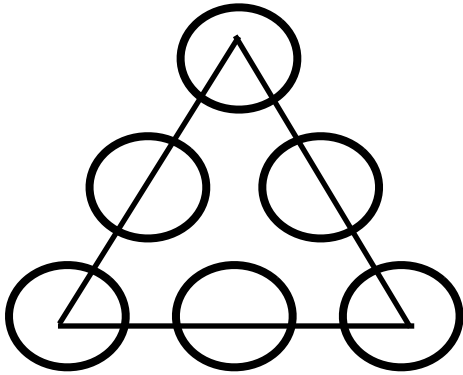
$$6 + \underline{\quad} + \underline{\quad}$$

$$6 + \underline{\quad} + \underline{\quad}$$

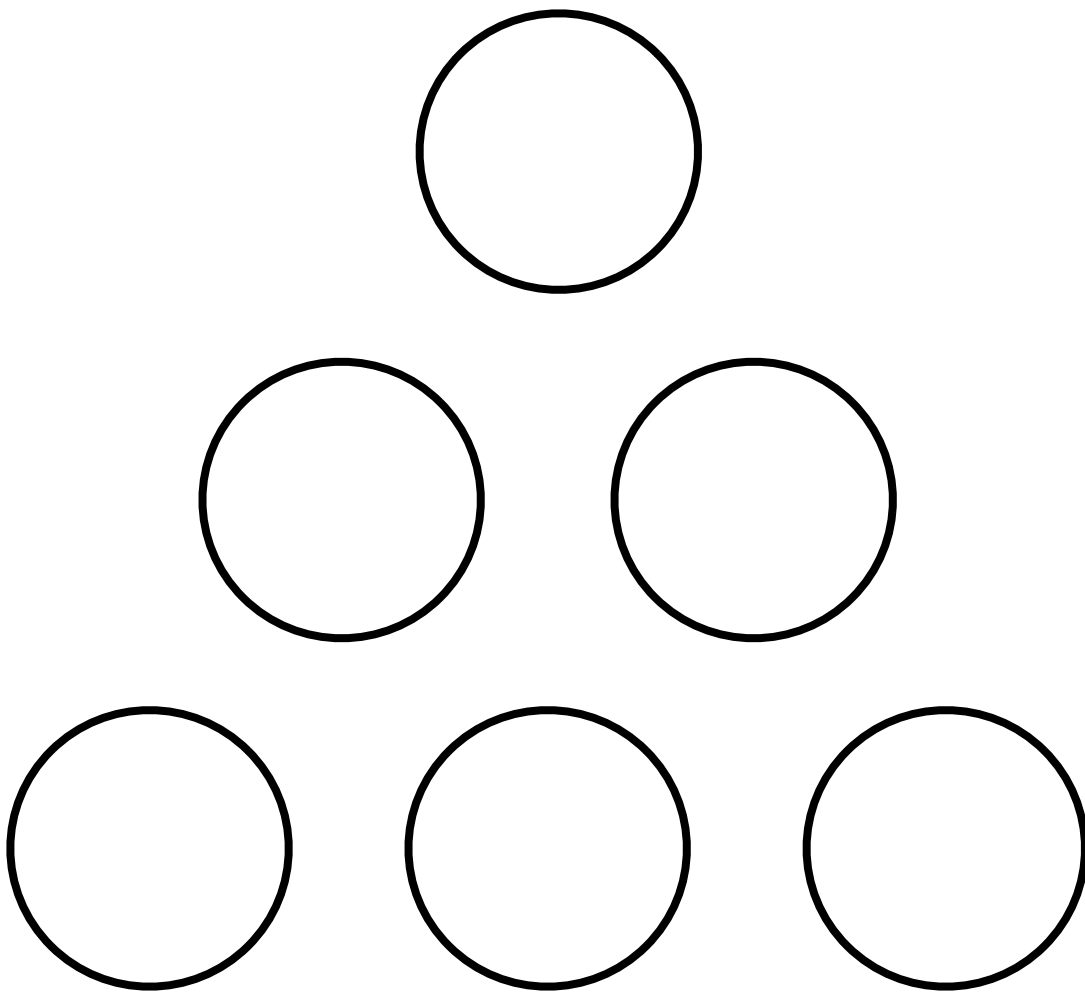
$$6 + \underline{\quad} + \underline{\quad}$$

$$6 + \underline{\quad} + \underline{\quad}$$

Perimeter Magic Triangle Sum to 12

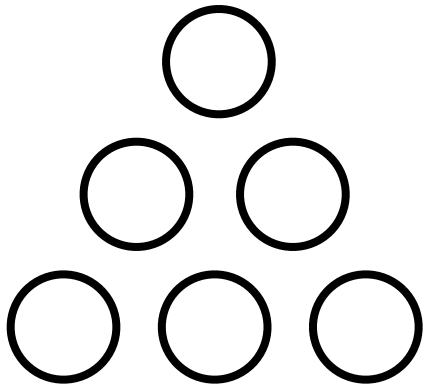


Arrange **each** of the **numbers from 1 to 6**
in the triangles
so that the numbers
in each row of 3 triangles
have a **Sum of 12.**

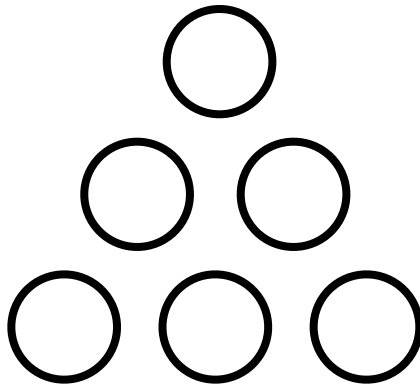


Perimeter Magic Triangle Sum to 12

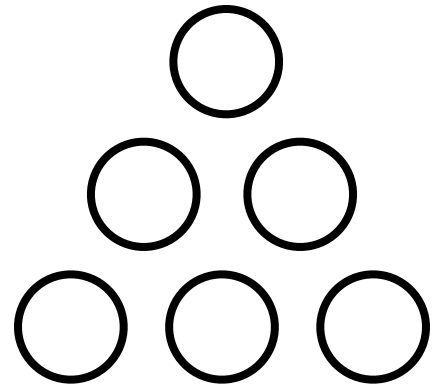
Find 3 **different** numbers from 1 to 6 that have a total of **12** and write them on the bottom row of the first picture. Then use the **remaining 3 numbers** to fill in the last 3 circles. See if the total of each row is 12. If it is not, keep trying different combinations until you find a way to use each of the numbers from 1 to 6 and get all 3 rows to total **12**?



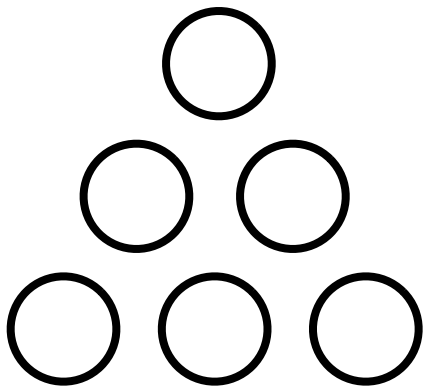
attempt 1



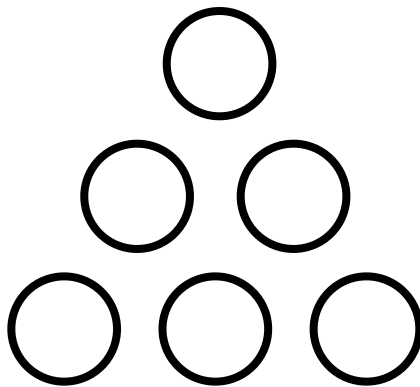
attempt 2



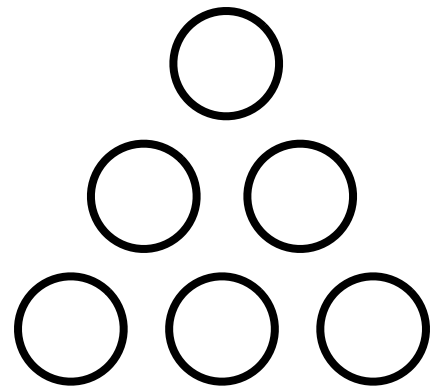
attempt 3



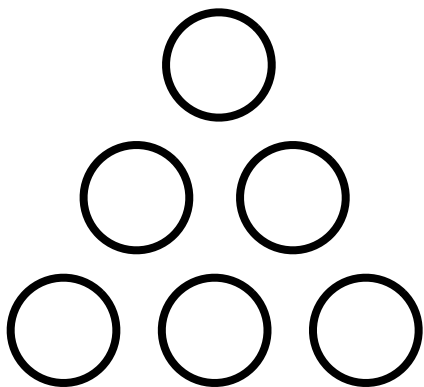
attempt 4



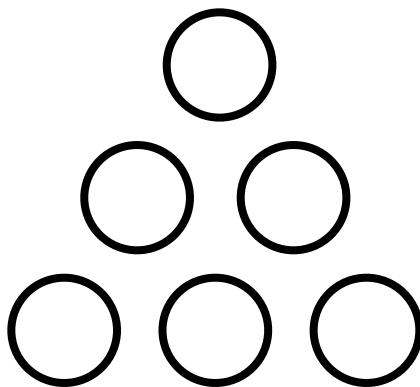
attempt 5



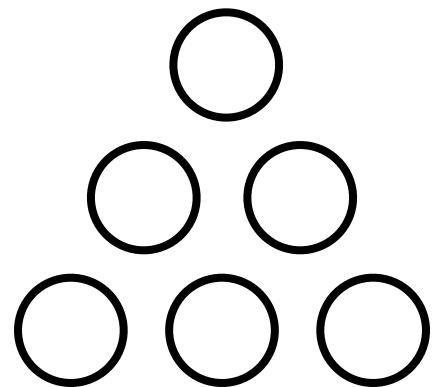
attempt 6



attempt 7



attempt 8



attempt 9

Perimeter Magic Triangle Sum to 12 help page

Find all the different ways that 3 numbers from 1 to 6 can be added to get a total of **12**. You cannot use the same number twice in any sum. The same 3 numbers can be used in different orders. EXAMPLE: $1 + 5 + 6 = 12$ and $1 + 6 + 5 = 12$ both work.

starts with a 1

$$1 + \underline{\quad} + \underline{\quad}$$

$$1 + \underline{\quad} + \underline{\quad}$$

starts with a 2

$$2 + \underline{\quad} + \underline{\quad}$$

$$2 + \underline{\quad} + \underline{\quad}$$

starts with a 3

$$3 + \underline{\quad} + \underline{\quad}$$

$$3 + \underline{\quad} + \underline{\quad}$$

starts with a 4

$$4 + \underline{\quad} + \underline{\quad}$$

$$4 + \underline{\quad} + \underline{\quad}$$

$$4 + \underline{\quad} + \underline{\quad}$$

$$4 + \underline{\quad} + \underline{\quad}$$

starts with a 5

$$5 + \underline{\quad} + \underline{\quad}$$

$$5 + \underline{\quad} + \underline{\quad}$$

starts with a 6

$$6 + \underline{\quad} + \underline{\quad}$$

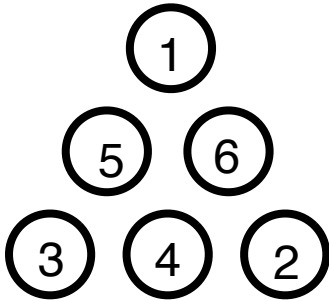
$$6 + \underline{\quad} + \underline{\quad}$$

$$6 + \underline{\quad} + \underline{\quad}$$

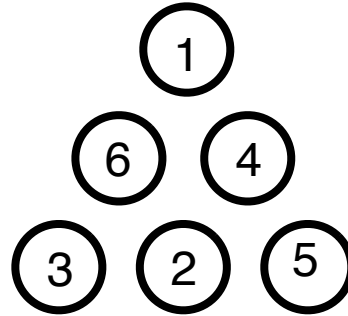
$$6 + \underline{\quad} + \underline{\quad}$$

Magic triangle solutions.

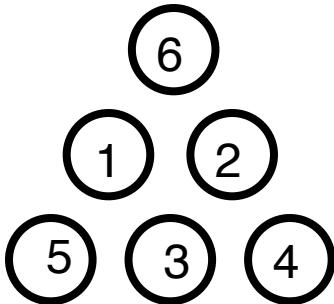
the magic total is 9



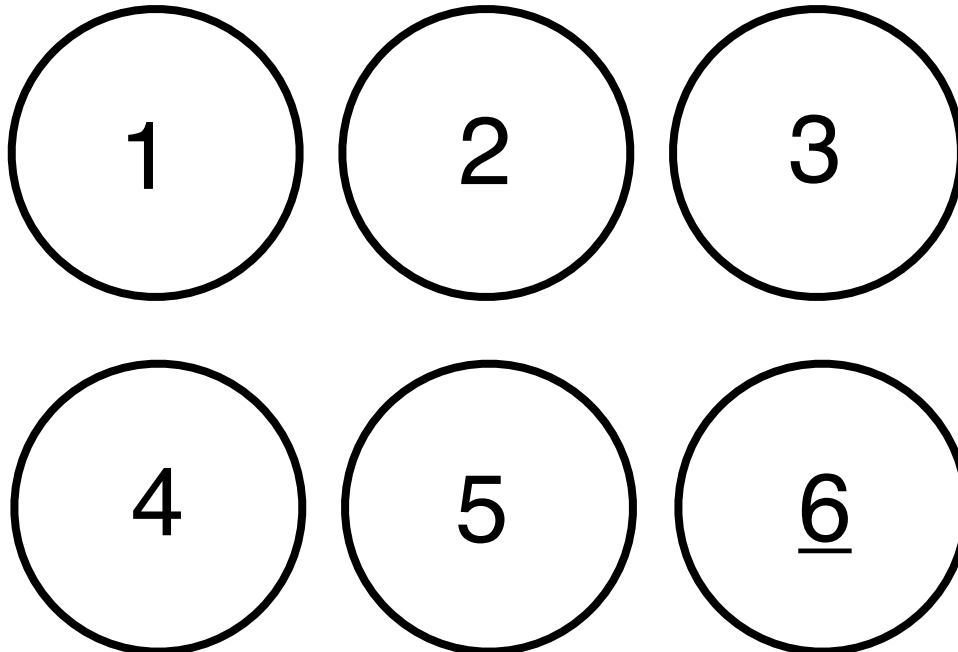
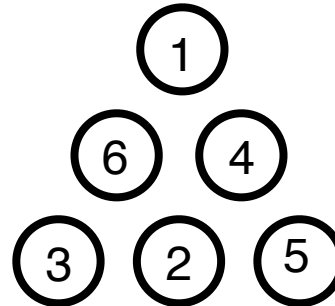
the magic total is 10



the magic total is 11



the magic total is 12



Notes to the teacher

Just having the student complete puzzle 1 is not the reason for having students work on this type of puzzle. The reason this set of puzzles have been used for over 100 years is the wealth of basic math and problem solving skills that can be used in finding the solution.

Many students start problem solving by trail and error. You would prefer that they try more efficient methods but as this may be the first puzzle of this type they have encountered then trail and error may be a good first step. The helper page for each puzzle is intended for this purpose. Students could put any three numbers that total 9 in the bottom row. They could then use the other 3 numbers to find the correct groupings for the other two rows.

The trial and error technique has a drawback. If the student does not record the failed efforts, they will repeat past failures until they luck out and find the solution or give up. The helper pictures allows them to record their attempts and avoids this issue. They will see the benefit of recording each effort even if they are using trail and error.

A much improved strategy is to look at all the possible sums at the start and see if you can get some insight from the list of sums.

You will need to provide direct instruction the first time they use this strategy.

This is not a “discover it on your own” strategy for most students. I let them find the solution by guess and check and then I see if there are different rotations of the same answer. I get the 3 solutions and we confirm that we all have the same solution.

I then ask the leading question: Are there other solutions or is this the only one?

I work at the front of the class. I have them start with 1 and see all the ways the other numbers may go with it to total 9. I then list 2 and do the same thing and the 3, 4, 5 and 6. make sure that see how we are using a pattern to get all the sums without missing any.

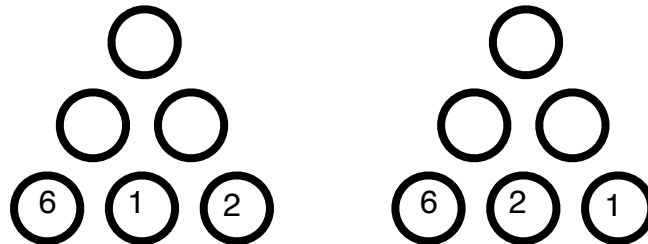
I help them see that many of the combinations are the same but in different order. **At this point I ask the leading question: What does it mean if a number is put in the vertex circle?**

The answer to that question is found in the work on the next page. This work that proves there is only one solution and what that solution must be.

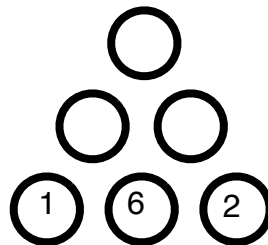
A list of all the possible ways 3 different numbers from 1 to 6 could total 9

$1 + 2 + 6$	$1 + 6 + 2$	$1 + 3 + 5$	$1 + 5 + 3$
$2 + 3 + 4$	$2 + 4 + 3$	$2 + 6 + 1$	$2 + 1 + 6$
$3 + 1 + 5$	$3 + 5 + 1$	$3 + 2 + 4$	$3 + 4 + 2$
$4 + 2 + 3$	$4 + 3 + 2$	$4 + 5 + 3$	$4 + 3 + 5$
$5 + 1 + 3$	$5 + 3 + 1$	$5 + 4 + 3$	$5 + 3 + 4$
$6 + 1 + 2$	$6 + 2 + 1$		

Look at the last 2 sums $6 + 2 + 1$ and $6 + 2 + 1$. If either one of them is used in the bottom row then 6 will be in the vertex circle.



That means there must be another sum with that has a 6 and two numbers different then a 1 and 2 to use in the row that will share the 6 at the vertex. The only sums with a 6 all have a 6, 2 and 1 so there are no other sums that are different than 6, 2 and 1. This means that the sums with 6, 2 and 1 must not have the 6 at the end. The 6 must be in the middle between a 1 and a 2

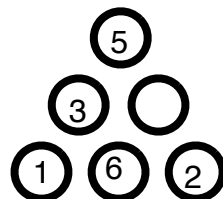


The 6, 2 and 1 have been used . We are now left with the following sets of 3 numbers and we need a set that has a 1 at the end

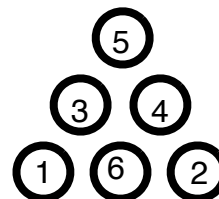
$1 + 3 + 5$	$1 + 5 + 3$	$2 + 3 + 4$	$2 + 4 + 3$	$3 + 1 + 5$	$3 + 5 + 1$
$3 + 2 + 4$	$3 + 4 + 2$	$4 + 2 + 3$	$4 + 3 + 2$	$4 + 5 + 3$	$4 + 3 + 5$
$5 + 1 + 3$	$5 + 3 + 1$	$5 + 4 + 3$	$5 + 3 + 4$		

The only set that works is a 1, 3 and 5

Our only choice is to put the 1, 3 and 5 in the left row



Our only choice is to put the 4 in the right row



We are done!!!
This,
or a rotation of
this,

Having the students list **all the ways** that 3 numbers selected from the numbers 1 to 6 can total 9 is a good activity. Some students can find the combinations faster than others but everyone can find all the combinations if they stay at it.

Discussing why changing the order of the sum does not change the sum adds the word commutative to their vocabulary.

Organizing the sums so that you can begin to look for patterns is the first step in almost every advanced problem solving task.

The solution to this puzzle is based on the fact that the order of the sum is important. The numbers in the circles at the vertex are used in two different sums. The numbers in the middle circles are only used in one sum. This key insight to this puzzle is that the order of the sum is important because the starting and ending numbers in the sums are in the vertex circles. These vertex numbers must also be in other sums along with different numbers.

The only way to get at the key insight is to look at all the sums and the relationships of the vertex numbers. Guess and check will help solve the puzzle but it will not lead to this key insight. The big secret we don't like to admit is that guess and check is almost always easier with simple problems than other strategies.

Who would use the second strategy when "guess and check" seems so much easier?

This is the big question!! The answer to the question is the key to how we need to approach problems solving. After we have used guess and check to solve easier version of the puzzle, we need to discuss the more work intensive strategy and use it to confirm the solution.

**We then need to immediately follow up
with a more complicated version of the same problem.**

In most cases students will approach the more complicated version with a mixture of guess and check and the second strategy just presented. The more complicated problem harder to solve with guess and check and most students move towards looking at the sums as part of the solution strategy. The key is to use a similar problem that has a more complicated solution

That is why the companion puzzle to this is the Magic Triangle with 4 numbers on a side for the numbers 1 to 9. Guess and check alone is not as efficient in the extended problem. Students that have solved the 3 numbers on a side puzzle will be challenged by the addition of the extra numbers. The basic issues that were the key to solving the 3 numbers on a side puzzle are still in play but the use of both strategies together will improve their ability to solve the puzzle.

Guess and Check may solve problems but it cannot provide the proof that it is the only solution. A more complete analysis is required.