Base 3 Number Prediction Cards

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Base 3: Predict a Number from 1 to 80

1	2	4	5	7	8
10	11	13	14	16	17
19	20	22	23	25	26
28	29	31	32	34	35
37	38	40	41	43	44
46	47	49	50	52	53
55	56	58	59	61	62
64	65	67	68	70	71
73	74	76	77	79	80

3	4	5	6	7	8
12	13	14	15	16	17
21	22	23	24	25	26
30	31	32	33	34	35
39	40	41	42	43	44
48	49	50	51	52	53
57	58	59	60	61	62
66	67	68	69	70	71
75	76	77	78	79	80

9	10	11	12	13	14
15	16	17	18	19	20
21	22	23	24	25	26
36	37	38	39	40	41
42	43	44	45	46	47
48	49	50	51	52	53
63	64	65	66	67	68
69	70	71	72	73	74
75	76	77	78	79	80

27	28	29	30	31	32
33	34	35	36	37	38
39	40	41	42	43	44
45	46	47	48	49	50
51	52	53	54	55	56
57	58	59	60	61	62
63	64	65	66	67	68
69	70	71	72	73	74
75	76	77	78	79	80

Base 3: Predict a Number from 1 to 80

Procedure: Cut out the 4 cards.

Ask a student to think of a number from 1 to 80 inclusive. You then hand the student the 4 cards . You ask the student to look at the cards and find the cards that have their number on it. Tell them that their number may not be on all the cards. Be sure to ask them to look closely at the cards so they do not miss a number on one of the cards. Ask the student to **hand you each card that has their number on it one card at a time and to tell you if the number they selected is red or black.** After the cards with their number on them have been given back and the color of the number has been stated you announce their exact number!

How it's done:

As they hand you a card find the **smallest number on the card that has the color the student stated their number was**. For these cards the smallest number is the first occurrence of the color on the card. Add up those numbers. The total will be the number that they thought of.

Example 1

The student picks 50 They hand you 4 cards

They hand you a card and their number is black. The smallest black number on that cards is 2.
They hand you a card and their number is red. The smallest red number on that cards is 3
They hand you a card and their number is black. The smallest black number on that cards is 18.
They hand you a card and their number is red. The smallest red number on that cards is 27.

Their number is 27 + 18 + 3 + 2 = 50

Example 2

The student picks 42 They hand you 3 cards

They hand you a card and their number is black. The smallest black number on that cards is 6.
They hand you a card and their number is red. The smallest red number on that cards is 9.
They hand you a card and their number is red. The smallest red number on that cards is 27.

Their number is 27 + 9 + 6 = 42

How does it work ?

We need to start with a smaller set of numbers and work with them first to develop an understanding of how the cards were developed. We can then extend the cards to work with more numbers. We will develop a set of cards for the numbers 1 to 26 base 10.

Base 3 notation for the numbers 1 to 26 base 10

Base 10	1	2	3	4	5	6	7	8	9
Base 3	001	002	010	011	012	020	021	022	100

Base 10	10	11	12	13	14	15	16	17	18
Base 3	101	102	110	111	112	120	121	122	200

Base 10	19	20	21	22	23	24	25	26	
Base 3	201	202	210	211	212	220	221	222	

Houston we have a problem !!!

We cannot use the cards in the same way we used base 2 cards. Look at the card for the 1's place value from the base 2 card.

Card 1			
1	3	5	7
9	11	13	15

There are only 2 possible digits that can go in the 1's place base 2, 0 or 1. If the number we selected is **NOT** on the 1's place card a 0 is put in the 1's place. If the number we selected **is** on the 1's place card a 1 is put in the 1's place.

What's the problem with base 3 ??

In base 3 there are 3 possible digits that can go in the 1's place: , 0 ,1 or 2. If the number selected is **NOT** on the 1's place card a 0 is put in the 1's place. But if the number we selected **is** on the 1's place card we do not know if a 1 or 2 should be placed in the 1's place.

How can we make a 1's place card that tells us which numbers have a 1 in the 1's place and which of the numbers have a 2 in the 1's place

The numbers 1 to 26 base 10 written in base 3

Base 10	1	2	3	4	5	6	7	8	9
Base 3	001	002	010	011	012	020	021	022	100

Base 10	10	11	12	13	14	15	16	17	18
Base 3	101	102	110	111	112	120	121	122	200

Base 10	19	20	21	22	23	24	25	26	
Base 3	201	202	210	211	212	220	221	222	

Notice that each base 3 number from 1 to 26 have 3 place values a 1's place a 3's place and a 9's place. You can use the digits 0, 1 or 2 in any of the place values.

How to make a card to find the number in the 1's place

We need two cards to let us know if a 1 or 2 are in the 1's place. We will call them Card 1A and Card 1B.

All of the numbers from 1 to 26 base 10 that have a 1 in the 1's place base 3 will to go on Card 1A. The upper left corner of Card 1A starts with a 1.

All of the numbers from 1 to 26 base 10 that have a 2 in the 1's place base 3 will to go on Card 1B. The upper left corner of Card 1B starts with a 2.

Any of the numbers from 1 to 26 base 10 have a 0 in the 1's place base 3 will not be listed on either card.

Using the tables of base 3 above make the 2 1's place cards.

Card 1A		
1	**4**	**7**
10	**13**	**16**
19	**22**	**25**

Card 1B		
2	**5**	**8**
11	**14**	**17**
20	**23**	**26**

Card 1A represents all the numbers that have a 1 in the 1's place base 3

Card 1B represents the numbers that have a 2 in the 1's place base 3

If the number is not on card 1A or card 1B then a zero is put in the 1's place.

The numbers 1 to 26 base 10 written in base 3

Base 10	1	2	3	4	5	6	7	8	9
Base 3	001	002	010	011	012	020	021	022	100

Base 10	10	11	12	13	14	15	16	17	18
Base 3	101	102	110	111	112	120	121	122	200

Base 10	19	20	21	22	23	24	25	26	
Base 3	201	202	210	211	212	220	221	222	

How to make a card to find the number in the 3's place

We need two cards to let us know if a 1 or 2 are in the 3's place. We will call them Card 2A and Card 2B.

All of the numbers from 1 to 26 base 10 that have a 1 in the 3's place base 3 will to go on Card 2A. The upper left corner of Card 2A starts with a 3. A 1 in the 3's place means the number is 3 base 10

All of the numbers from 1 to 26 base 10 that have a 2 in the 3's place base 3 will to go on Card 2B. The upper left corner of Card 1B starts with a 6. A 2 in the 3's place means the number has 2 threes and is equal to 6 base 10.

Any of the numbers from 1 to 26 base 10 have a 0 in the 9's place base 3 will not be listed on either card.

Card 2A		
3	**4**	**5**
12	**13**	**14**
21	**22**	**23**

Card 2B		
6	**7**	**8**
15	**16**	**17**
24	**25**	**26**

Card 2A represents all the numbers that have a 1 in the 3's place base 3

Card 2B represents the numbers that have a 2 in the 3's place base 3

If the number is not on card 2A or card 2B then a zero is put in the 3's place.

The numbers 1 to 26 base 10 written in base 3

Base 10	1	2	3	4	5	6	7	8	9
Base 3	001	002	010	011	012	020	021	022	100

Base 10	10	11	12	13	14	15	16	17	18
Base 3	101	102	110	111	112	120	121	122	200

Base 10	19	20	21	22	23	24	25	26	
Base 3	201	202	210	211	212	220	221	222	

How to make a card to find the number in the 3's place

We need two cards to let us know if a 1 or 2 are in the 9's place. We will call them Card 3A and Card 3B.

All of the numbers from 1 to 26 base 10 that have a 1 in the 9's place base 3 will to go on Card 3A. The upper left corner of Card 3A starts with a 9. A 1 in the 9's place means the number is 9 base 10

All of the numbers from 1 to 26 base 10 that have a 2 in the 9's place base 3 will to go on Card 3B. The upper left corner of Card 3B starts with an 18. A 2 in the 9's place means the number has 2 nines and is equal to 10 base 10

Any of the numbers from 1 to 26 base 10 have a 0 in the 3's place base 3 will not be listed on either card.

Card 3A		
9	**10**	**11**
12	**13**	**14**
15	**16**	**17**

Card 3B		
18	**19**	**20**
21	**22**	**23**
24	**25**	**26**

Card 3A represents all the numbers that have a 1 in the 9s place base 3

Card 3B represents the numbers that have an 2 in the 9's place base 3

If the number is not on card 3A or card 3B then a zero is put in the 9's place.

The numbers 1 to 26 base 10 written in in base 3

Base 10		How many 9's 3's 1's	card 3 9's	card 2 3's	card 1 1's	base 3 9 3 1
1	0 + 0 + 1	9(0) + 3(0) + 1(1)	0	0	1	0 0 1
2	0 + 0 + 2	9(0) + 3(0) + 1(2)	0	0	2	0 0 2
3	0 + 3 + 0	9(0) + 3(1) + 1(0)	0	1	0	0 1 0
4	0 + 3 + 1	9(0) + 3(1) + 1(1)	0	1	1	0 1 1
5	0 + 3 + 2	9(0) + 3(1) + 1(2)	0	1	2	0 1 2
6	0 + 6 + 0	9(0) + 3(2) + 1(0)	0	2	0	0 2 0
7	0+ 6 + 1	9(0) + 3(2) + 1(1)	0	2	1	0 2 1
8	0 + 6 + 2	9(0) + 3(2) + 1(2)	0	2	2	0 2 2
9	9 + 0 + 0	9(1) + 3(0) + 1(0)	1	0	0	1 0 0
10	9 + 0 + 1	9(1) + 3(0) + 1(1)	1	0	1	1 0 2
11	9 + 0 + 2	9(1) + 3(0) + 1(2)	1	0	2	1 0 2
12	9 + 3 + 0	9(1) + 3(1) + 1(0)	1	1	0	1 1 0
13	9 + 3 + 1	9(1) + 3(1) + 1(1)	1	1	1	1 1 1
14	9 + 3 + 2	9(1) + 3(1) + 1(2)	1	1	2	1 1 2
15	9 + 6 + 0	9(1) + 3(2) + 1(0)	1	2	0	1 2 0
16	9 + 6 + 1	9(1) + 3(2) + 1(1)	1	2	1	1 2 1
17	9 + 6 + 2	9(1) + 3(2) + 1(2)	1	2	2	1 2 2
18	18 + 0 + 0	9(2) + 3(0) + 1(0)	2	0	0	2 0 0
19	18 + 0 + 1	9(2) + 3(0) + 1(1)	2	0	1	2 0 1
20	18 + 0 + 2	9(2) + 3(0) + 1(2)	2	0	2	2 0 2
21	18 + 3 + 0	9(2) + 3(1) + 1(0)	2	1	0	2 1 0
22	18 + 3 + 1	9(2) + 3(1) + 1(1)	2	1	1	2 1 1
23	18 + 3 + 2	9(2) + 3(1) + 1(2)	2	1	2	2 1 2
24	18 + 6 + 0	9(2) + 3(2) + 1(0)	2	2	0	2 2 0
25	18 + 6 + 1	9(2) + 3(2) + 1(1)	2	2	1	2 2 1
26	18 + 6 + 2	9(2) + 3(2) + 1(2)	2	2	2	2 2 2

What table should the numbers 1 to 26 be placed on ?

Base 10	base 3 9 3 1	card 3B 2 9's 18	card 3A 1 9's 9	card 2B 2 3's 6	card 2A 1 3's 3	card 1B 1 1's 2	card 1A 1 1's 1
1	0 0 1						yes
2	0 0 2					yes	
3	0 1 0				yes		
4	0 1 1				yes		yes
5	0 1 2				yes	yes	
6	0 2 0			yes			
7	0 2 1			yes			yes
8	0 2 2			yes		yes	
9	1 0 0		yes				
10	1 0 1		yes				yes
11	1 0 2		yes			yes	
12	1 1 0		yes		yes		
13	1 1 1		yes		yes		yes
14	1 1 2		yes		yes	yes	
15	1 2 0		yes	yes			
16	1 2 1		yes	yes			yes
17	1 2 2		yes	yes		yes	
18	2 0 0	yes					
19	2 0 1	yes					yes
20	2 0 2	yes				yes	
21	2 1 0	yes			yes		
22	2 1 1	yes			yes		yes
23	2 1 2	yes			yes	yes	
24	2 2 0	yes		yes			
25	2 2 1	yes		yes			yes
26	2 2 2	yes		yes		yes	

The 6 cards created for the base 10 number form 1 to 26 in base 3

Card 1A		
1	4	7
10	13	16
19	22	25

Card 1B		
2	5	8
11	14	17
20	23	26

Card 1A represents all the numbers that have a 1 in the 1's place base 3

Card 1B represents the numbers that have a 2 in the 1's place base 3

If the number is not on card 1A or card 1B then a zero is put in the 1's place.

Card 2A		
3	4	5
12	13	14
21	22	23

Card 2B		
6	7	8
15	16	17
24	25	26

Card 2A represents all the numbers that have a 1 in the 3's place base 3

Card 2B represents the numbers that have a 2 in the 3's place base 3

If the number is not on card 2A or card 2B then a zero is put in the 3's place.

Card 3A		
9	10	11
12	13	14
15	16	17

Card 3B		
18	19	20
21	22	23
24	25	26

Card 3A represents all the numbers that have a 1 in the 9s place base 3

Card 3B represents the numbers that have an 2 in the 9's place base 3

If the number is not on card 3A or card 3B then a zero is put in the 9's place.

The trick works best if the cards are not numbered. The next pages contain the cards you will use when you perform the trick with students.

Base 3: Predict a number from 1 to 26

1	4	7
10	13	16
19	22	25

2	5	8
11	14	17
20	23	26

3	4	5
12	13	14
21	22	23

6	7	8
15	16	17
24	25	26

9	10	11
12	13	14
15	16	17

18	19	20
21	22	23
24	25	26

Base 3: Predict a number from 1 to 26

Procedure: Cut out the 6 cards.

Ask a student to think of a number from 1 to 26 inclusive. You then hand the student all 6 cards . You ask the student to look at the cards and hand you all the cards that have their number on it. Tell them that their number may not be on all the cards. Be sure to ask them to look closely at the cards so they do not miss a number on one of the cards. After the cards with their number on them have been given back you announce their exact number!

How it's done:

Find the **smallest number** on each of the cards given to you. For these cards the smallest number is in the top left square. Add up those numbers. The total will be the number that they thought of.

Example 1:

The student picks 20 They hand you 2 cards.

The smallest number on one of the cards is 2
The smallest number on a second card is 10

Their number is 18 + 2 = 20

Example 2:

The student picks 13. They hand you 3 cards.

The smallest number on one of the cards is 1.
The smallest number on a second card is 3.
The smallest number on a third card is 9.

Their number is 9 + 3 + 1 = 13

Extension

It is possible to condense the information on 2 cards onto 1 card for each place value.

Make the number on card 1A red. Leave the number on card 1B black

Card 1A		
1	4	7
10	13	16
19	22	25

Card 1B		
2	5	8
11	14	17
20	23	26

Merge the numbers onto 1 card.

Card 1					
1	2	4	5	7	8
10	11	13	14	16	17
19	20	22	23	25	26

I chose to put the numbers in order.
The smallest red number is 1 and the smallest black number is 2.
This card contains all the information as cards 1A and 1B but the student needs to tell you the color of his number if it is on the card instead of saying yes to Card 1A or Card 1B

Make the number on card 2A red. Leave the number on card 2B black

Card 2A		
3	4	5
12	13	14
21	22	23

Card 2B		
6	7	8
15	16	17
24	25	26

Merge the numbers onto 1 card.

Card 2					
3	4	5	6	7	8
12	13	14	15	16	17
21	22	23	24	25	26

I chose to put the numbers in order.
The smallest red number is 3 and the smallest black number is 6.
This card contains all the information as cards 2A and 2B but the student needs to tell you the color of his number if it is on the card instead of saying yes to Card 2A or Card 2B

Make the number on card 2A red. Leave the number on card 2B black

Card 3A		
9	10	11
12	13	14
15	16	17

Card 3B		
18	19	20
21	22	23
24	25	26

Merge the numbers onto 1 card.

Card 3					
9	10	11	12	13	14
15	16	17	18	19	20
21	22	23	24	25	26

I chose to put the numbers in order.
The smallest red number is 9 and the smallest black number is 18.
This card contains all the information as cards 3A and 3B but the student needs to tell you the color of his number if it is on the card instead of saying yes to Card 3A or Card 3B

Base 3 : Predict a number from 1 to 26

Card 1					
1	2	4	5	7	8
10	11	13	14	16	17
19	20	22	23	25	26

Card 2					
3	4	5	6	7	8
12	13	14	15	16	17
21	22	23	24	25	26

Card 3					
9	10	11	12	13	14
15	16	17	18	19	20
21	22	23	24	25	26

Note: The addition of a color for each row allows us to condense the 6 cards onto 3 cards. The key numbers used for the place value are now based on the lowest number with that color

Base 3: Predict a number from 1 to 26

1	2	4	5	7	8
10	11	13	14	16	17
19	20	22	23	25	26

3	4	5	6	7	8
12	13	14	15	16	17
21	22	23	24	25	26

9	10	11	12	13	14
15	16	17	18	19	20
21	22	23	24	25	26

Base 3: Predict a number from 1 to 26

Procedure: Cut out the 3 cards.

Ask a student to think of a number from 1 to 26 inclusive. You then hand the student the 3 cards . You ask the student to look at the cards and find the cards that have their number on it. Tell them that their number may not be on all the cards. Be sure to ask them to look closely at the cards so they do not miss a number on one of the cards. Ask the student to **hand you each card that has their number on it one card at a time and to tell you if the number they selected is red or black.** After the cards with their number on them have been given back and the color of the number has been stated you announce their exact number!

How it's done:

As they hand you a card find the **smallest number on the card that has the color the student stated their number was**. For these cards the smallest number is in the top left square. Add up those numbers. The total will be the number that they thought of.

Example 1

The student picks 17 They hand you 3 cards

They hand you a card and their number is black. The smallest black number on that cards is 2.
They hand you a card and their number is black. The smallest black number on that cards is 6.
They hand you a card and their number is red. The smallest red number on that cards is 9.

Their number is 9 + 6 + 2 = 17

Example 2

The student picks 7 They hand you 2 cards

They hand you a card and their number is red. The smallest red number on that cards is 1.
They hand you a card and their number is black. The smallest black number on that cards is 6.

Their number is 6 + 1 = 7

Card 1					
1	2	4	5	7	8
10	11	13	14	16	17
19	20	22	23	25	26

Card 2					
3	4	5	6	7	8
12	13	14	15	16	17
21	22	23	24	25	26

Card 3					
9	10	11	12	13	14
15	16	17	18	19	20
21	22	23	24	25	26

Can you condense the cards even more?

You could condense the information on the 3 cards onto one card but this would require 6 different colors. FInding 6 colors that are that different in appearance poses a minor problem. It would also require a card with 54 numbers on it. Many of the numbers would be repeated in as many as three colors. At that point it is very probable that the student would not be able to find all the occurrences of their number and the trick would not work very well. Sometimes one improvement in a given area like reducing the number of cards used causes an increase in difficulty in another area. The three cards with 2 colors on each card may be the best balance.