Binary Search Cards

Discovering how they work

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Hiding the obvious

Math magic is all about hiding the obvious. If I gave you **Card 1 below** and said my number is in the first row and first column you would easily see the number is a 1. If I gave you **Card 1 below** and said my number is in the third row and third column you would easily see the number is a 12. This is a fast an easy way to identify the position of a number on a grid. The game battleship uses this technique to place a bomb in a given square.

Card 1

	C1	C2	C3	C4
R1	**1**	2	3	4
R2	5	6	7	8
R3	9	10	11	**12**
R4	13	14	15	16

Finding a number on a card by asking what row and column it is in is not much of a trick. For a trick to impress a student we must hide the obvious reference to a row and column. These hole cards do just that. The cards find the row and column of the number you select without making it obvious that is what they are doing.

It is a good trick if we have a mysterious set cards with numbers in a random looking order and holes in the card at seemingly random places. It gets even better if the front and the back of the cards both have numbers on them and some of the back sides of the cards have numbers upside down from their front sides. Cards that do all that would help produce a great trick.

What could be better.

For a trick to be good it must be fairly easy to explain how it works. It would be better if it were fairly easy for a student to make a different set of cards of their own. Since this is a math magic trick it would be great if the math behind the trick was based on an important concept the students study in junior high and high school. It would be almost impossible to expect that this math also extends into computer programming and is used in first year computer science classes in college.

This trick and these cards do all of the above

It will take a few pages to cover all the information mentioned above. We will start with the basics of the trick and develop the way the cards are made. We will extend this to a new set of cards the same size that we stared with. We will then extend the trick to a larger set of cards.

Along the way we will discover the connection these cards have to base 2 cards. We will finish with the connection these cards have to a sorting algorithm in computer science and refer to the basic “bubble sort” that all first year computer science students learn to program. You can stop at an early point and enjoy just making the cards or stay with us from the start to finish and see what the entire trip looks like.

Lets start the trick without the hole cards and then see how the basic trick works

Pick any number on the card from 1 to 16

My number is 1

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

The trick depends on finding the answers to 4 questions. After each question is answered we block out or **eliminate the numbers that you cannot have selected.** After we get the answers to the four questions below we will have eliminated all but 1 number in a given row and column.

1. Is your number on the top half or the bottom half of the card? Top. Eliminate **rows 3 and 4**
2. Is your number on the right half or the left half of the card? Left. Eliminate **columns 3 and 4**
3. Is your number in an odd or an even column? Odd. Eliminate **columns 2 and 4**
4. Is your number in an odd or an even row? Odd. Eliminate **rows 2 and 4**

We have eliminated Rows 2, 3 and 4. We have eliminated columns 2, 3 and 4. Your number is in row 1 column 1. Your number is 1

The elimination looks like this using the grid

I am in the top half

	C1	C2	C3	C4
R1	1	2	3	4
R2	5	6	7	8
R3				
R4				

eliminate the numbers in the bottom half

I am in the left half

	C1	C2	C3	C4
R1	1	2		
R2	5	6		
R3				
R4				

eliminate the numbers on the right side

I am in an odd column

	C1	C2	C3	C4
R1	1			
R2	5			
R3				
R4				

eliminate the numbers in col. 2 and col. 4

I am in an odd row

	C1	C2	C3	C4
R1	**1**			
R2				
R3				
R4				

eliminate the numbers in row 2 and row 4

	C1	C2	C3	C4
R1	**1**			
R2				
R3				
R4				

We have eliminated Rows 2, 3 and 4. We have eliminated columns 2, 3 and 4. Your number is in row 1 column 1. Your number is 1

Lets start the trick without the hole cards and then see how the basic trick works

Pick any number on the card from 1 to 16

My number is 1

1	2	3	4
5	6	7	8
9	10	11	**12**
13	14	15	16

The trick depends on finding the answers to 4 questions. After each question is answered we block out or **eliminate the numbers that you cannot have selected.** After we get the answers to the four questions below we will have eliminated all but 1 number in a given row and column.

1. Is your number on the top half or the bottom half of the card? Bottom. Eliminate **rows 1 and 2**
2. Is your number on the right half or the left half of the card? Right. Eliminate **columns 1 and 2**
3. Is your number in an odd or an even column? Even. Eliminate **columns 1 and 3**
4. Is your number in an odd or an even row? Odd. Eliminate **rows 2 and 4**

We have eliminated Rows 1, 2 and 4. We have eliminated columns 1, 2 , 3. Your number is in row 3 column 4. Your number is 12.

I am in the bottom half

	C1	C2	C3	C4
R1				
R2				
R3	9	10	11	12
R4	13	14	15	16

eliminate the numbers in the top half

I am on the right side

	C1	C2	C3	C4
R1				
R2				
R3			11	12
R4			15	16

eliminate the numbers on the left side

I am in an even column

	C1	C2	C3	C4
R1				
R2				
R3				12
R4				16

eliminate the numbers in col. 1 and col. 3

I am in an odd row

	C1	C2	C3	C4
R1				
R2				
R3				**12**
R4				

eliminate the numbers in row 2 and row 4

	C1	C2	C3	C4
R1				
R2				
R3				**12**
R4				

We have eliminated Rows 1, 2 and 4. We have eliminated columns 1, 2 , 3. Your number is in row 3 column 4. Your number is 12.

How do I make cards that will block all the numbers on the base card except one number?

We need to create hole cards that block the bottom half of the base card, the right half of the base card the even rows of the base card and the even columns of the base card. We also need the student to select the correct hole cards from a larger set of hole cards based on the number that they selected.

We will create a set of hole cards that block ll the numbers on the base card except 1

Base Card

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Step 1. We need to make a card that blocks out the numbers on the bottom half and leaves the numbers on the top half visible when it is laid on top of the base card.

Card 1A

cut out this area			
1	2	3	4
5	6	7	8

Making Card 1A

Write the 8 numbers form the top half of the base card in the 8 squares in the bottom half of a square card. Cut out the marked area leaving a hole in the top half of the hole card. If Card 1A is laid on top of the base card the numbers on the bottom half of the base card are blocked from view but you can still see the numbers in the top half of the base card

Card 1B

9	10	11	12
13	14	15	16
cut out this area			

Making Card 1B

Write the 8 numbers from the bottom half of the base card in the 8 squares in the top half of a square card. Cut out the marked area leaving a hole in the bottom half of the hole card. If Card 1B is laid on top of the base card the numbers on the top half of the base card are blocked from view but you can still see the numbers in the bottom half of the base card

Ask the student if Card 1A or Card 1B has their number on it. It is Card 1A. Have them put card 1A on top of the base card. With Card 1A on top of the base card you can see the 8 numbers from the base card (shown in white). The numbers shown in gray are from Card 1A. **You have eliminated from view all but the 8 numbers in white from the base card.**

Base Card

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Card 1A

cut out this area			
1	2	3	4
5	6	7	8

Card 1A lying on top of the base card

1	2	3	4
5	6	7	8
1	2	3	4
5	6	7	8

After step one

Card 1A lying on top of the **base card**

1	2	3	4
5	6	7	8
1	2	3	4
5	6	7	8

Step 2. We need to make a card that blocks out the numbers on the right half and leaves the numbers on the left half visible when it is laid on top of the base card.

Card 2A

cut out this area	1	2
	5	6
	9	10
	13	14

Making Card 2A

Write the 8 numbers from the left half of the base card in the 8 squares on the right half of a square card. Cut out the marked area leaving a hole in the left half of the hole card. If Card 2A is laid on top of the base card the numbers on the right half of the base card are blocked from view but you can still see the numbers on the left half of the base card

Card 2B

3	4	cut out this area
7	8	
11	12	
15	16	

Making Card 2B

Write the 8 numbers from the right half of the base card in the 8 squares on the left half of a square card. Cut out the marked area leaving a hole in the bottom half of the hole card. If Card 2B is laid on top of the base card the numbers on the left half of the base card are blocked from view but you can still see the numbers on the right half of the base card

Ask the student if Card 2A or Card 2B has their number on it. It is Card 2A. Have them put that card on top of stack which has the base card on the bottom and Card 1A on top of it. With Card A on top of Card 1A on top of the base card you can see the 4 numbers from the base card (shown in white) The numbers shown in gray are from Card 1A and Card 2A **You have eliminated form view all but the 4 numbers in white from the base card.**

Card1A lying on top of the base card

1	2	3	4
5	6	7	8
1	2	3	4
5	6	7	8

Card 2A

cut out this area	1	2
	5	6
	9	10
	13	14

Card 2A lying on top of Card 1A lying on top of the base card

1	2	1	2
5	6	5	6
1	2	9	10
5	6	13	14

After step two

Card 2A lying on top of Card 1A lying on top of the base card

1	2	1	2
5	6	5	6
1	2	9	10
5	6	13	14

Step 3. We need to make a card that blocks out the numbers on the even Column(Column 2 and Column 4) and leaves the numbers on the odd Column (Column 1 and Column 3) visible when it is laid on top of the base card.

Card 3A

cut out	1	cut out	3
	5		7
	9		11
	13		15

Making Card 3A

Write the 4 numbers from Column 1 of the base card in the 4 squares of Column 2 of a square card. Write the 4 numbers from Column 3 of the base card in the 4 squares of Column 4 of the square card. Cut out the marked areas leaving 2 holes in the hole card.

Card 3B

2	cut out	4	cut out
6		8	
10		12	
14		16	

Making Card 3B

Write the 4 numbers from Column 2 of the base card in the 4 squares of Column 1 of a square card. Write the 4 numbers from Column 4 of the base card in the 4 squares of Column 3 of the square card. Cut out the marked areas leaving 2 holes in the hole card.

Ask the student if Card 3A or Card 3B has their number on it. It is Card 3A. Have them put that card on top of stack which has the base card on the bottom and Cards 1A and 2A on top of it. You can see 2 numbers from the base card (shown in white) The numbers shown in gray are from Card 1A and Card 2A **You have eliminated form view all but the 2 numbers in white from the base card.**

Card 2A lying on top of Card 1A lying on top of the base card

1	2	1	2
5	6	5	6
1	2	9	10
5	6	13	14

Card 3A

cut out	1	cut out	3
	5		7
	9		11
	13		15

Card 3A lying on top of Card 2A lying on of Card 1A lying on top of the base card

1	1	1	3
5	5	5	7
1	9	9	11
5	13	13	15

After step three

Card 3A on top of Card 2A on of Card 1A on top of the base card

1	1	1	3
5	5	5	7
1	9	9	11
5	13	13	15

Step 4. We need to make a card that blocks out the numbers on the even rows (Row 2 and Row 4) and leaves the numbers on the odd rows (Row 1 and Row 3) visible when it is laid on top of the base card.

Card 4A

cut out			
1	2	3	4
cut out			
9	10	11	12

Making Card 4A

Write the 4 numbers from Row 1 of the base card in the 4 squares of Row 2 of a square card. Write the 4 numbers from Row 3 of the base card in the 4 squares of Row 4 of the square card. Cut out the marked areas leaving 2 holes in the hole card.

Card 4B

5	6	7	8
cut out			
13	14	15	16
cut out			

Making Card 4B

Write the 4 numbers from Row 2 of the base card in the 4 squares of Row 1 of a square card. Write the 4 numbers from Row 4 of the base card in the 4 squares of Row 3 of the square card. Cut out the marked areas leaving 2 holes in the hole card.

Ask the student if Card 4A or Card 4B has their number on it. It is Card 4A. Have them put that card on top of stack which has the base card on the bottom and Cards 1A and 2A and 3A on top of it. You can see 1 number from the base card (shown in white) The numbers shown in gray are from Cards 1A, 2A and 3A **You can only see 1 number from the base card. All the others are form the hole cards.**

Card 3A on top of Card 2A on top of Card 1A on top of the base card

1	1	1	3
5	5	5	7
1	9	9	11
5	13	13	15

Card 4A

cut out			
1	2	3	4
cut out			
9	10	11	12

Card 4A on top of Card 3A on top of Card 2A on top of Card 1A on top of the base card

1	1	1	3
5	5	5	7
1	9	9	11
5	13	13	15

We are done. The single number visible from the base card is the number they selected. The number they selected is 1

Summery of the 4 hole cards

We need to make a card that blocks out the bottom half of the base card and a card that blocks out the top half of the base card.

Card 1A

cut out this area			
1	2	3	4
5	6	7	8

Card 1B

9	10	11	12
13	14	15	16
cut out this area			

If the students number is from 1 to 8 they will pick Card 1A. Card 1A blocks out the bottom half of the base card.

If the students number is from 9 to 13 they will pick Card 1B. Card 1B blocks out the top half of the base card.

We need to make a card that blocks out the right half of the base card and a card that blocks out the left half of the base card.

Card 2A

cut out this area	1	2
	5	6
	9	10
	13	14

Card 2B

3	4	cut out this area
7	8	
11	12	
15	16	

If the students number is from the left side of the base card they will pick Card 2A. Card 2A blocks out the right side of the base card.
If the students number is from the right side of the base card they will pick Card 2B. Card 2B blocks out the left side of the base card.

We need to make a card that blocks out the even columns of the base card and a card that blocks out the odd columns of the base card.

Card 3A

cut out	1	cut out	3
	5		7
	9		11
	13		15

Card 3B

2	cut out	4	cut out
6		8	
10		12	
14		16	

If the students number is from an odd column of the base card they will pick Card 3A. Card 3A blocks out the even columns of the base card.
If the students number is from an even column of the base card they will pick Card 3B. Card 3B blocks out the odd columns of the base card.

We need to make a card that blocks out the even rows of the base card and a card that blocks out the odd rows of the base card.

Card 4A

cut out			
1	2	3	4
cut out			
9	10	11	12

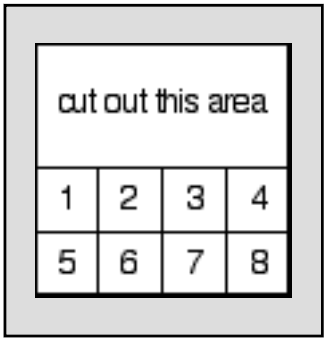
Card 4B

5	6	7	8
cut out			
13	14	15	16
cut out			

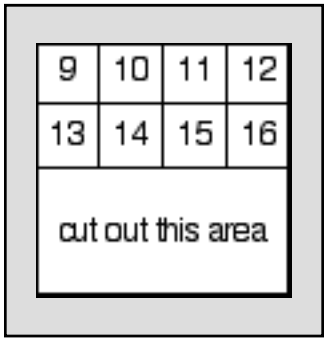
If the students number is from an odd roe of the base card they will pick Card 4A. Card 4A blocks the even rows of the base card.
If the students number is from an even row of the base card they will pick Card 4B. Card 4B blocks the even rows of the base card.

You will need a border around each card. This is required so that the card is rigid and so that the holes have 4 solid pieces of paper around them. The **final cards used for the trick**

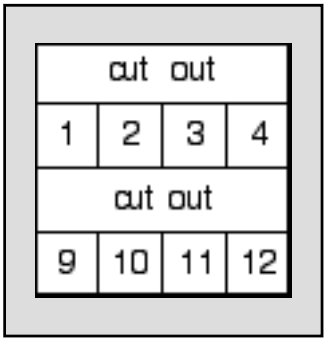
Card 1A



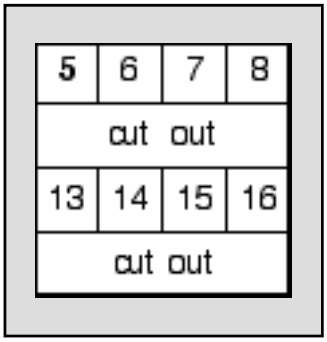
Card 1B



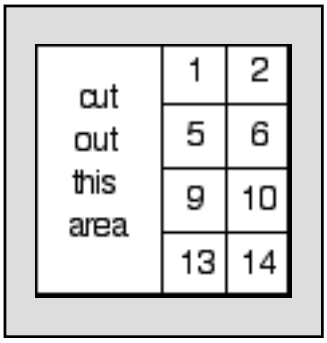
Card 4A



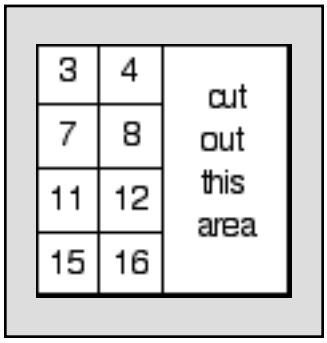
Card 4B



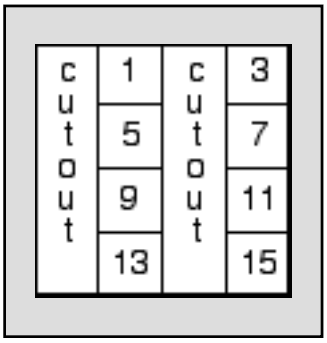
Card 2A



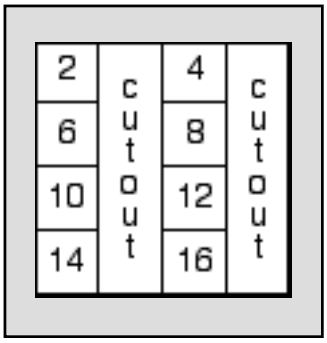
Card 2B



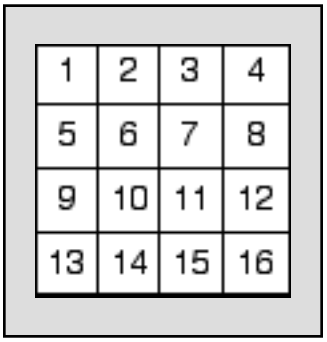
Card 3A



Card 3B



Base Card



Pick a number form 1 to 16

The student picks a 1. They are asked to find the cards that have their number on it and stack them on top of the base card.

1. They pick Card 1A? It blocks the bottom 2 rows of the base card. Eliminate **rows 3 and 4**
2. They pick Card 2A? It blocks the 2 right columns of the base card Eliminate **columns 3 and 4**
3. They pick Card 3A? It blocks the 2 even columns of the base card. Eliminate **columns 2 and 4**
4. They pick Card 4A? It blocks the 2 even rows of the base card Eliminate **rows 2 and 4**

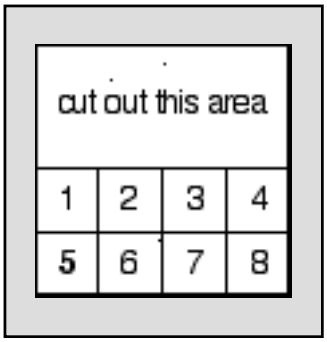
When you look at the base card covered with the 4 hole cards you will see that only one number form the base card can be seen. It is the number one. We a white base card. If we had a red base card the 1 would stand out as the only number on the base card that is not covered by the hole cards.

Reducing the 8 cards to 4 cards

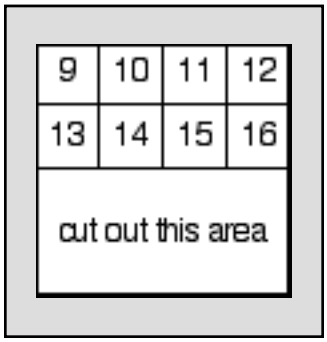
You can reduce the 8 hole cards to 4 cards each with a front and back. To do this requires gluing each set of hole cards back to back with the cutout are in the same location so that when you can cut out the area on one side the hole will be correctly positioned for both cards.

Cards 1A and 1B can be glued together but Card 1B will need to be flipped over so the cut out areas are in the same location. This means one side of the card has numbers that face up and the other side faces down. This is also true for Card 4A and Card 4B.

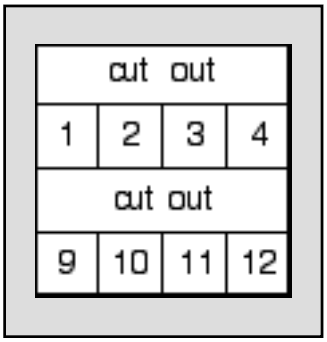
Card 1A



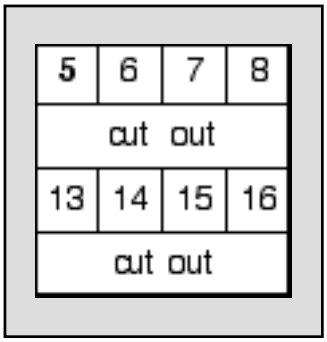
Card 1B



Card 4A

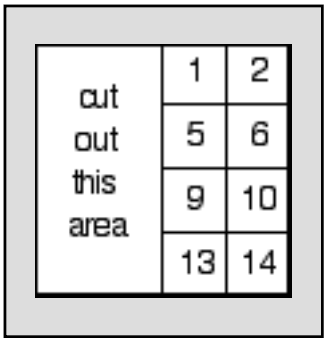


Card 4B

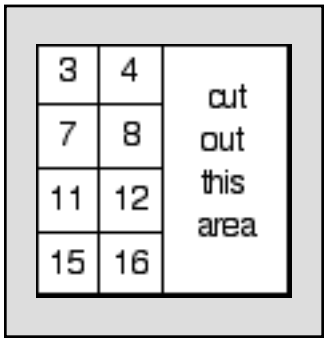


Cards 2A and 2B can be glued together by putting the cards back to back with the numbers on both sides facing upwards This is also true for Card 3A and Card 3B.

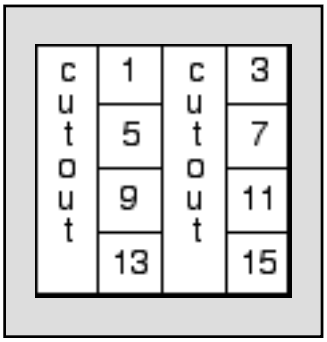
Card 2A



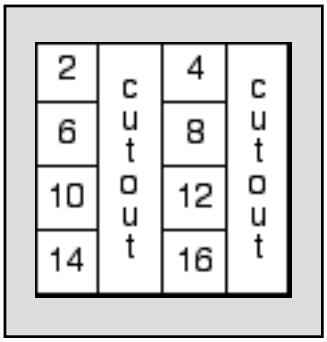
Card 2B



Card 3A



Card 3B



I gave you the cards in a larger form on the following pages. The first version has the 8 hole cards and the base card to print out. Cut out and glue the cards together in the correct orientations and then cut out the holes. If you have put the cards together the hole is in the correct place on the front and back.

Save time with a 2 sided printer

For those with a printer that will print on both sides (2 sided copies) I provide 4 pages that can be printed so the printer prints 4 hole cards with the correct front and back for you. Its a time saver if you have the correct printer.

Base 10, Base 2 and Base hole card?

Many students are familiar with writing base 10 numbers in base 2

Base 10 place values _____ _____ _____ _____
1000's place 100"splace 10 place units place

Base 2 place values _____ _____ _____ _____
8's place 4"splace 2's place 1'splace

Base 2 only allows for the digits 0 or 1. We call such a number base a binary number system.

The 4 sets of hole cards form a binary number system

The 4 sets of hole cards can be used as place values to write binary numbers. Each binary number represents a location on the base card. If the base card contains the numbers to 16 then each number from 1 to 16 can be written as a binary number with the sets of hole cards as place values.

The 16 numbers on the base card can be written in base hole card

	C1	C2	C3	C4
R1	1	2	3	4
R2	5	6	7	8
R3	9	10	11	12
R4	13	14	15	16

each base card number

1A 2A 3A 4A	1A 2A 3B 4A	1A 2B 3A 4A	1A 2B 3B 4A
1A 2A 3A 4B	1A 2A 3B 4B	1A 2B 3A 4B	1A 2B 3B 4B
1B 2A 3A 4A	1B 2A 3B 4A	1B 2B 3A 4A	1B 2B 3B 4A
1B 2A 3A 4B	1B 2A 3B 4B	1B 2B 3A 4B	1B 2B 3B 4B

is on 4 hole cards

TH LS R1 C1	TH LS R1 C2	TH RS R1 C3	TH RS R1 C4
TH LS R2 C1	TH LS R2 C2	TH RS R2 C3	TH RS R2 C4
BH LS R3 C1	BH LS R3 C2	BH RS R3 C3	BH RS R3 C4
BH LS R4 C1	BH LS R4 C2	BH RS R4 C3	BH RS R2 C4

based on being in the top-bottom half
the left or right half
an odd or even column
or an odd or even row

We read the digits in a number right to left so the binary number base describes which side of the 4 hole cards a base number is on.

4A odd row = 0 4B even row = 1	3A odd column = 0 3B even column = 1	2A left half = 0 2B right half = 1	1A top half = 0 1B bottom half = 1

The first digit in base hole card tells you if the number on the base card is on hole card 1A or 1B
The second digit in base hole card tells you if the number on the base card is on hole card 2A or 2B
The third digit in base hole card tells you if the number on the base card is on hole card 3A or 3B
The fourth digit in base hole card tells you if the number on the base card is on hole card 4A or 4B

	C1	C2	C3	C4
R1	1	2	3	4
R2	5	6	7	8
R3	9	10	11	12
R4	13	14	15	16

1A 2A 3A 4A	1A 2A 3B 4A	1A 2B 3A 4A	1A 2B 3B 4A
1A 2A 3A 4B	1A 2A 3B 4B	1A 2B 3A 4B	1A 2B 3B 4B
1B 2A 3A 4A	1B 2A 3B 4A	1B 2B 3A 4A	1B 2B 3B 4A
1B 2A 3A 4B	1B 2A 3B 4B	1B 2B 3A 4B	1B 2B 3B 4B

4A odd row = 0
4B even row = 1

3A odd column = 0
3B even column = 1

2A left half = 0
2B right half = 1

1A top half = 0
1B bottom half = 1

Example 1: 8 on the base card is on hole cards 4B 3A, 2B, 1A

1	1	1	0	base hole card
4B	3B	2B	1A	

Example 2: 11 on the base card is on hole cards 1B, 2B, 3A 4A

0	0	1	1	base hole card
4B	3A	2B	1B	

The 16 base card numbers written in the binary base hole card

1 base card = 0 0 0 0 base hole card

2 base card = 0 1 0 0 base hole card

3 base card = 0 0 1 0 base hole card

4 base card = 0 1 1 0 base hole card

5 base card = 0 0 0 1 base hole card

6 base card = 1 1 0 0 base hole card

7 base card = 1 0 1 0 base hole card

8 base card = 1 1 1 0 base hole card

9 base card = 0 0 0 1 base hole card

10 base card = 0 1 0 1 base hole card

11 base card = 0 0 1 1 base hole card

12 base card = 0 1 1 1 base hole card

13 base card = 1 0 0 1 base hole card

14 base card = 1 1 0 1 base hole card

15 base card = 1 0 1 1 base hole card

16 base card = 1 1 1 1 base hole card

Binary Search

In computer science, a binary search or **half-interval search** algorithm finds the **position** of a specified key value within an array. The binary search halves the number of items to check with each iteration. In the first step, the algorithm determines which half of the array the key value is a member of . The algorithm then looks at this new array (1/ 2 of the old one) and determines which half of the new array the key value is a member of. In the each preceding step, the algorithm keeps developing an array that is half the old array and determining which half of the new array the search value is a member of. This process continues until you get to one position in the array. The number in that position is the key value you were searching for.

We wish to find the location of a selected number on the sorted base card.

We do this by breaking the locations into 2 groups and ask which group the number is in.

We then break the new group we got from the first search into 2 groups and ask which group the number is in.

We then break the new group we got from the last search into 2 groups and ask which group the number is in.

This process continues until you get to one position in the array. The number in that position is the key value you were searching for. **This process is called a binary search.**

Our first search question tells us if our number is in the top or bottom half of the locations

Is it on the top half or the bottom half of the locations? Answer: Top half. We look for the position of the number in the top half of the locations on the base card.

Our next search question tells us if our number is in the right or left half of the locations

Is it on the left half or the right half of the locations? Answer: Left half. We look for the position of the number in the top left half of the locations of the base card

Our next search question tells us if our number is in the odd or even half of the columns.

Is it in odd or even columns? Answer: Odd column. We look for the position of the number in an odd columns in the top left half of the base card

Our next search question tells us if our number is in the odd or even half of the rows.

Is it in odd or even row? Answer: Odd row. We look for the position of the number in an odd row in odd column in the top left half of the base card.

The number we are looking for is in the location that is the top left part of the array in the 1'st column and 1'st row.

This is a binary search program written in Java

```
public int binary Search ( List < Card > cards, Card key)

{   int mid = (cards.size()) / 2;
   if(cards.size() == 1) {
      if( key.equals( cards.get ( 0 ) ) ) {
         return 0;  }
        }
   else {
      if ( key.equals(cards.get (mid) ) ) {
         return mid;
      }
      else if ( key.compare To (cards.get ( mid ) ) == - 1) {
         return binarySearch(cards.subList(0, mid), key);
      }
      else if ( key.compareTo ( cards.get ( mid ) ) ==  1) {
         return mid + 1 + binarySearch (cards.subList (mid + 1, cards.size( )) , key);
      }
   }
   return -1;
}
```

Binary Cards require special array sizes

Binary search cards are limited to cards where the number of items in a row or column is a power of 2. dimension must be a power of 2. A normal binary search takes half the array and then half of that half and then half of the half and so on. The only numbers that can be continually halved and still get whole number are the powers of 2: 2 , 4 , 8 , 16 ... If you start with a row of 16 positions you get 8 rows where your number could be after the first halving. There are 4 rows where your number could be after the second halving. There are 2 rows where your number could be after the third halving. There is one row where your number could be after the last halving.

Computer programs can do binary searches on any size array. Programs can be designed to handle the cases where tone of the dimensions is an odd number. If you start with 36 elements the binary search will then state which half of the 36 numbers your number is in. It will then state which of the groups of 18 your number is in . It will then divide the 18 elements in to tow half's and determine which of the groups of 9 your number is in. At this point a program cam adjust to the odd numbers and continue a search using 2 even sets and one number extra. The Binary search cards represent a mechanical system that requires every time we have the set we get a new set that is an even number. This requirement limits the type of array we can use for a Binary Card system.

The number of positions in any rows or column must be a power of 2. The set of cards I refer to in this set use 4 or eight positions in any row or column. Versions 1 , 2 and 6 use 4 by 4 cards. Version 3 uses 8 by 8 cards. Version 4 uses 4 by 8 cards. Version 5 uses 2 by 4 cards.